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Nonlinear Vibration of Microbeams under Magnetic Field Using the Modified Couple Stress Theory

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

In this work, nonlinear vibration of microbeams under magnetic field is studied. The equation governing motion of microbeam resting on a linear elastic layer and under magnetic field is derived by using the modified couple stress theory and Hamilton's principle. Amplitude–frequency relationships of microbeams with pinned–pinned and clamped–clamped end conditions are obtained in closed-forms by using the equivalent linearization method with a weighted averaging. Accuracy of the present solution is verified by comparising the obtained solution with previous solutions. Efects of the material length scale parameter, the stiffness coefficient of the linear foundation and the magnetic field on the frequency ratios of microbeams are investigated in this paper.

Keywords: Modified couple stress theory; nonlinear vibration; microbeams; equivalent linearization method; weighted averaging.

1 Introduction

Micro/Nano-beams have many applications, especially in Micro/Nano-electromechanical systems (MEMS/NEMS) such as sensors, nanowires, micro-actuators, micro-switches and atomic force microscopes

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[1-5]. Unlike structures at the macro-scale where the microstructure size-dependency can be neglected; for structures at the micro-scale, material size dependence is very important and cannot be neglected. The classical elastic continuum theory is not suitable to describe behaviors of micro-/nano-structures, thus some higher-order elastic theories have been developed to describe the size-dependent behavior of micro-/nano-structures such as the nonlocal elasticity theory proposed by Eringen [6], the strain gradient theory introduced by Aifantis [7], the couple stress theory proposed by Toupin, Mindlin and Tiersten [8,9] and the nonlocal strain gradient theory proposed by Lim, Zhang and Reddy [10].

The couple stress theory [8,9], besides the two classical elastic constants (Lamé constants), contains two material constants (the material length scale parameters). Because of difficultity in applying, the couple stress theory was modified by Yang, Chong, Lam and Tong in 2002 [11] by introducing an applicable theory with only one additional material constant was considered, this theory was called the modified couple stress theory (MCST). MCST is more useful than the classical one, thus this theory is widely used by researchers. To date, many works related to micro/nano-beams using MCST have been published. Nateghi et al. analyzed buckling of functionally graded materials (FGM) microbeams based on MCST [12]. Based on a new unified beam theory and the modifed couple stress theory, bending and vibration behaviors of FGM microbeams were investigated by Şimşek and Reddy [13]. Static and dynamic beharvior of the third-order shear deformation FGM microbeam based on MCST was studied by Salamattalab et al. [14]. Free vibration of nonuniform microbeams was investigated by Khaniki and Hashemi using MCST [15]. Based on MCST and Euler–Bernoulli beam theory together with the von-Kármán's nonlinear strain–displacement relationship, Şimşek studied the static and nonlinear vibration of microbeams resting on the nonlinear elastic foundation [16]. In the work of Jam et al. [17], MCST was used to analyze nonlinear free vibration behavior of microbeams resting on the viscoelastic layer. And, based on MCST, Hieu [18] investigated the postbuckling and nonlinear free vibration of microbeams resting on three nonlinear elastic layers.

Vibration responses of beams and tubes under magnetic force also attract many authors. Based on nonlocal elasticity theory, vibration of double-walled carbon nanotubes under the longitudinal magnetic field was investigated by Murmu et al. [19]. Nonlinear free vibration of nonlocal elasticity nanobeams under magnetic field was studied by Chang [20]. Sun et al. used the nonlocal elasticity theory to analyze nonlinear vibration of buckled nanobeams under longitudinal magnetic field [21]. Effect of longitudinal magnetic field on vibration response of a sing-walled carbon nanotube embedded in viscoelastic medium was investigated in the work of Zhang et al. [22]. And, influence of magnetic field on size sensitivity of nonlinear vibration of embedded nanobeams was studied by Zhao et al. using the nonlocal Timoshenko beam theory [23].

However, according to authors' knowledge, there has no published work on the vibration of microbeams resting on the elastic foundation under magnetic force based on MCST. Thus, in this work, based on MCST and the Von-Kármán's assumptions about strain-displacement relationships, we focus on analyzing nonlinear free vibration of microbeams resting on linear elastic medium under effect of magnetic force. The equivalent linearization method (ELM) with a weighted averaging [24,25] is used to obtain approximate frequencies of microbeams with pinned-pinned (P-P) and clamped-clamped (C-C) end conditions. Accuracy of the current solution is verified by comparising the obtanied solution with the published solution and the numerical solution using the 4^{th}-order Runge-Kutta method. Effects of the material length scale parameter, the magnetic field and the coefficient of the linear elastic layer on nonlinear vibration behavior of microbeams are investigated in this paper.

2 Governing Equation

2.1 The modified couple stress theory

The strain energy for an isotropic linearly elastic material occupying a volume Ω according to the modifed couple stress theory [11] is given as:

$$U_1 = \int_\Omega \frac{1}{2}(\boldsymbol{\sigma}:\boldsymbol{\varepsilon} + \mathbf{m}:\boldsymbol{\chi})d\Omega = \int_\Omega \left[\mu\left(\boldsymbol{\varepsilon}:\boldsymbol{\varepsilon} + l^2\boldsymbol{\chi}:\boldsymbol{\chi}\right) + \frac{1}{2}\lambda(tr\boldsymbol{\varepsilon})^2 \right] d\Omega. \quad (1)$$

where $\boldsymbol{\sigma}$, $\boldsymbol{\varepsilon}$, $\mathbf{m}$ and $\boldsymbol{\chi}$ are the symmetric part of the Cauchy stress tensor, the strain tensor, the deviatoric part of the couple stress tensor and the symmetric curvature tensor, respectively; l is the material length scale parameter; μ and λ are Lamé constants.

The kinematic relations are given as:

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\nabla\mathbf{u} + (\nabla\mathbf{u})^T\right], \quad (2)$$

$$\boldsymbol{\chi} = \frac{1}{2}\left[\nabla\boldsymbol{\omega} + (\nabla\boldsymbol{\omega})^T\right], \quad (3)$$

where $\mathbf{u}$ and $\boldsymbol{\omega}$ are the displacement and rotation vectors, respectively, which are described as follows:

$$\boldsymbol{\omega} = \frac{1}{2} curl\,\mathbf{u}. \quad (4)$$

Expressions of tensors $\boldsymbol{\sigma}$ and $\mathbf{m}$ are described as:

$$\boldsymbol{\sigma} = \lambda tr(\boldsymbol{\varepsilon})\mathbf{I} + 2\mu\boldsymbol{\varepsilon}, \quad (5)$$

$$\mathbf{m} = 2l^2\mu\boldsymbol{\chi}. \quad (6)$$

Lamé constants λ and μ can be expressed as the elasticity modulus E and Poisson's ratio ν as follows:

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}, \quad \mu = \frac{E}{2(1+\nu)}. \quad (7)$$

2.2 The Governing Equation for Microbeam

An isotropic microbeam resting on a linear elastic foundation with the spring constant k_L of the Winkler elastic mediun is considered in Fig. 1. The microbeam of length L and cross-section dimension $b \times h$ is subjected to the longitudinal magnetic field.

The displacement field based on Euler–Bernoulli beam theory takes a following form:

$$u_x(x,z,t) = u(x,t) - z\frac{\partial w(x,t)}{\partial x}, \quad (8)$$

$$u_y(x,z,t) = 0, \quad (9)$$

$$u_z(x,z,t) = w(x,t), \quad (10)$$

here u_x, u_y and u_z are the displacements along x, y and z directions, respaectively; u and w are the axial and transverse displacements, respectively, of any point on the neutral axis of microbeam.

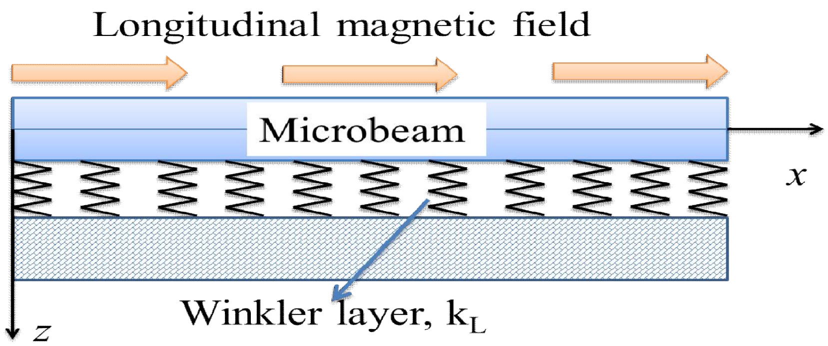

Fig. 1. Schematic of a microbeam resting on the linear elastic medium under the longitudinal magnetic field

The strain–displacement relationship based on the Von-Kármán's assumptions are expressed as:

$$\varepsilon_{xx} = \frac{\partial u(x,t)}{\partial x} + \frac{1}{2}\left(\frac{\partial w(x,t)}{\partial x}\right)^2 - z\frac{\partial^2 w(x,t)}{\partial x^2}; \varepsilon_{yy} = \varepsilon_{zz} = \varepsilon_{xy} = \varepsilon_{xz} = \varepsilon_{yz} = 0. \quad (11)$$

The rotation vector **ω** has the components given by:

$$\omega_x = 0,\ \omega_y = -\frac{\partial w(x,t)}{\partial x},\ \omega_z = 0. \quad (12)$$

And, the components of the symmetric curvature tensor $\boldsymbol{\chi}$ are given as:

$$\chi_{xy} = -\frac{1}{2}\frac{\partial^2 w(x,t)}{\partial x^2},\ \chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{xz} = \chi_{yz} = 0. \quad (13)$$

Using Hamilton's principle

$$\delta\int_0^t \left[T + W_{ext} - U_1 - U_2\right]dt = 0, \quad (14)$$

where T, W_{ext}, U_1 and U_2 are the kinetic energy, the work done by the external forces, the strain energy given in Eq. (1) and the strain energy induced by the linear elastic layer, respectively, the governing equation of motion of microbeam based on MCST can be obatined as follows:

$$\left(EI + \mu A l^2\right)\frac{\partial^4 w}{\partial x^4} - \left[\frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + k_L w + \rho A\frac{\partial^2 w}{\partial t^2} = F. \quad (15)$$

In Eq. (15), $I = bh^3/12$ is the inertia moment of the cross-section, $A = bh$ is the area of the cross-section, ρ is the the mass density of the microbeam, and F is the Lorentz force due to the longitudinal magnetic field. The Lorentz force along the z direction is denined as [20,21]:

$$F = \eta A H_x^2 \frac{\partial^2 w}{\partial x^2}. \quad (16)$$

where η is the magnetic permeability, H_x is the component of the longitudinal magnetic field.

Substituting Eq. (16) into Eq. (15), the equation of motion of micro beam becomes:

$$\left(EI + \mu A l^2\right)\frac{\partial^4 w}{\partial x^4} - \left[\frac{EA}{2L}\int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx\right]\frac{\partial^2 w}{\partial x^2} + k_L w + \rho A \frac{\partial^2 w}{\partial t^2} - \eta A H_x^2 \frac{\partial^2 w}{\partial x^2} = 0. \quad (17)$$

Introducing the dimensionless parameters:

$$\overline{x} = \frac{x}{L},\ \overline{w} = \frac{w}{r},\ t = \overline{t}\sqrt{\frac{\rho A L^4}{EI}},\ \kappa = \frac{l}{h},\ \Xi = 1 + \frac{6\kappa^2}{1+\nu},\ K_L = \frac{k_L L^4}{EI},\ H = \frac{\eta A L^2}{EI} H_x^2. \quad (18)$$

where $r = \sqrt{I/A}$ is the radius of gyration of the cross-section of the microbeam.

Considering Eq. (18), Eq. (17) can be written in the dimensionless form as follows:

$$\Xi \frac{\partial^4 \overline{w}}{\partial \overline{x}^4} - \left[\frac{1}{2}\int_0^1 \left(\frac{\partial \overline{w}}{\partial \overline{x}}\right)^2 d\overline{x}\right]\frac{\partial^2 \overline{w}}{\partial \overline{x}^2} + K_L \overline{w} - H \frac{\partial^2 \overline{w}}{\partial \overline{x}^2} + \frac{\partial^2 \overline{w}}{\partial \overline{t}^2} = 0. \quad (19)$$

The displacement function $\overline{w}(\overline{x}, \overline{t})$ is assumed to be expressed as:

$$\overline{w}(\overline{x}, \overline{t}) = V(\overline{t})\phi(\overline{x}), \quad (20)$$

where $V(\overline{t})$ is the unknown time-dependent function and $\phi(\overline{x})$ is the basis function satisfying the kinematic boundary conditions. The basis functions can be chosen as follows:

+ For P-P microbeam:

$$\phi(\overline{x}) = \sin(\pi \overline{x}). \quad (21)$$

+ For C-C microbeam:

$$\phi(\overline{x}) = \frac{1}{2}\left[1 - \cos(2\pi \overline{x})\right]. \quad (22)$$

Using Galerkin method, we substitute Eq. (20) into Eq. (19) then multiplying both sides of the obtained equation with $\phi(\overline{x})$ and integrating this equation over the domain (0,1), we get:

$$\ddot{V}(\bar{t})+\gamma_1 V(\bar{t})+\gamma_2 V^3(\bar{t})=0. \quad (23)$$

where

$$\gamma_1 = K_L - H\frac{\int_0^1 \phi''\phi d\bar{x}}{\int_0^1 \phi^2 d\bar{x}} + \Xi\frac{\int_0^1 \phi^{(4)}\phi d\bar{x}}{\int_0^1 \phi^2 d\bar{x}}, \quad (24)$$

$$\gamma_2 = -\frac{\frac{1}{2}\int_0^1\left[\int_0^1 (\phi')^2 d\bar{x}\right]\phi''\phi d\bar{x}}{\int_0^1 \phi^2 d\bar{x}}. \quad (25)$$

Assuming that microbeam has the following initial conditions:

$$V(0)=\alpha, \quad \dot{V}(0)=0, \quad (26)$$

where α is the dimensionless maximum amplitude of oscillation.

3 Solution Procedure

ELM with a weighted averaging [24, 25] will be used to find approximate solution of Eq. (23). Based on ELM proposed by Caughey [29], the equivalent linearization method with a weighted averaging has improved the disadvantages of the linearization method equivalent with the classical averaging. Some strongly nonlinear oscillations have been analyzed by applying this method [18, 25-28].

First, we introduce to the linearized form of the nonlinear equation (23) as follows:

$$\ddot{V}(\bar{t})+\omega^2 V(\bar{t})=0. \quad (27)$$

In Eq. (27), ω is known as the frequency of oscillation, it can be determined by using the mean square criterion which minimizes the error between two equations (23) and (27):

$$e^2(V)=\left(\gamma_1 V+\gamma_2 V^3-\omega^2 V\right)^2 \rightarrow \underset{\omega^2}{Min}. \quad (28)$$

From condition:

$$\frac{\partial\left\langle e^2(Q)\right\rangle}{\partial\omega^2}=0, \quad (29)$$

we get:

$$\omega^2=\frac{\gamma_1\left\langle V^2\right\rangle+\gamma_2\left\langle V^4\right\rangle}{\left\langle V^2\right\rangle}. \quad (30)$$

In Eq. (30), the symbol $\langle \square \rangle$ denotes the time – averaging operator in classical meaning [30]:

$$\langle V(\bar{t})\rangle = \lim_{T\to+\infty}\frac{1}{T}\int_0^T V(\bar{t})d\bar{t}. \quad (31)$$

For a ω-frequency function , the averaging process is taken during one period T, namely:

$$\langle V(\omega\bar{t})\rangle = \frac{1}{T}\int_0^T V(\omega\bar{t})dt = \frac{1}{2\pi}\int_0^{2\pi} V(\tau)d\tau, \quad \tau=\omega\bar{t}. \quad (32)$$

The averaging values in Eqs. (31) and (23) are called the classical averaging values. In this work, we use the weighted averaging [24, 25] to determine averaging values in Eq. (30). According to Anh's idea [24], the constant coefficient $1/T$ in Eqs. (31) and (32) is replaced by a weighted coefficient function $h(\bar{t})$. Thus, we get the weighted averaging value:

$$\langle V(\bar{t})\rangle_w = \int_0^{+\infty} h(\bar{t})V(\bar{t})d\bar{t}, \quad (33)$$

where the weighted coefficient function $h(\bar{t})$ satisfies the following condition:

$$\int_0^{+\infty} h(\bar{t})d\bar{t} = 1. \quad (34)$$

In this work, we use a specific form of the weighted coefficient function as follows [24]:

$$h(\bar{t}) = s^2\omega\bar{t}\,e^{-s\omega\bar{t}}, \quad (35)$$

where s is a positive constant. Eq. (33) will take the form of Eqs. (31) and (32) as s=0. The solution of Eq. (27) is given as:

$$Q(\bar{t}) = \alpha\cos(\omega\bar{t}) \quad (36)$$

With the periodic solution in Eq. (36), the averaging values $\langle V^2\rangle$ and $\langle V^4\rangle$ in Eq. (30) can be determined by using Eq. (33) with the weighted coefficient function given in Eq. (35) and Laplace transform as follows:

$$\begin{aligned}\langle V^2\rangle_w &= \langle \alpha^2\cos^2(\omega\bar{t})\rangle_w = \int_0^{+\infty}\alpha^2 s^2\omega\bar{t}\,e^{-s\omega\bar{t}}\cos^2(\omega\bar{t})d\bar{t}\\ &= \int_0^{+\infty}\alpha^2 s^2\tau\,e^{-s\tau}\cos^2(\tau)d\tau = \alpha^2\frac{s^4+2s^2+8}{(s^2+4)^2},\end{aligned} \quad (37)$$

$$\begin{aligned}\langle V^4\rangle_w &= \langle \alpha^4\cos^4(\omega\bar{t})\rangle_w = \int_0^{+\infty}\alpha^4 s^2\omega\bar{t}\,e^{-s\omega\bar{t}}\cos^4(\omega\bar{t})d\bar{t}\\ &= \int_0^{+\infty}\alpha^2 s^2\tau\,e^{-s\tau}\cos^4(\tau)d\tau = \alpha^4\frac{s^8+28s^6+248s^4+416s^2+1536}{(s^2+4)^2(s^2+16)^2}.\end{aligned} \quad (38)$$

Substituting Eqs. (37) and (38) into Eq. (30), we obtain the approximate frequency of oscillation:

$$\omega_{NL} = \sqrt{\gamma_1 + \gamma_2 \alpha^2 \frac{s^8 + 28s^6 + 248s^4 + 416s^2 + 1536}{(s^4 + 2s^2 + 8)(s^2 + 16)^2}}. \quad (39)$$

The approximate frequency ω_{NL} in Eq. (39) depends on both the initial amplitude *α* and the parameter *s*. With $s = 2$, we obtain the approximate frequency:

$$\omega_{NL} = \sqrt{\gamma_1 + 0.72\gamma_2 \alpha^2}. \quad (40)$$

Thus, the approximate solution of oscillation is:

$$V(\bar{t}) = \alpha \cos\left(\sqrt{\gamma_1 + 0.72\gamma_2 \alpha^2}\,\bar{t}\right). \quad (41)$$

Using the basis functions $\phi(\bar{x})$ given in Eqs. (21) and (22), and performing the integrations in Eqs. (24) and (25), we get the approximate frequencies:

+ For P-P microbeam:

$$\omega_{NL} = \sqrt{(K_L + \pi^2 H + \pi^4 \Xi) + 0.72\left(\frac{\pi^4}{4}\right)\alpha^2}. \quad (42)$$

+ For C-C microbeam:

$$\omega_{NL} = \sqrt{(K_L + \frac{4}{3}\pi^2 H + \frac{16}{3}\pi^4 \Xi) + 0.72\left(\frac{\pi^4}{3}\right)\alpha^2}. \quad (43)$$

4 Numerical Results and Discussions

Accuracy of the current solution is verified by comparing this solution with the published solution. The current frequencies are compared with the approximate frequencies achieved by Sun et al. [21]. Using the Multiple Scales Lindstedt-Poincare (MSLP) method and the nonlocal elasticity theory, Sun et al. [21] obtained the approximate frequency of nanobeam under magnetic field. Comparison is presented in Table 1 with noted that the material length scale parameter *l/h* and the linear elastic foundation parameter K_L are set to zero. We can see a very good agreement between the two approximate frequencies from this Table.

Fig. 2 shows the time history of responses for P-P and C-C microbeams are compared with the ones obtained by the 4th-order Runge-Kutta method. Again accuracy of the current solution can be observed.

Effects of the material length scale parameter *l/h*, the magnetic field and the Winkler parameter K_L of the elastic layer on the nonlinear frequency (ω_{NL}) and the frequency ratio (ω_{NL} / ω_L) of microbeam are investigated in the below part. Noted that the linear frequency of microbeam can be achieved by letting the initial amplitude to be zero ($\alpha = 0$). We can see from Figs. 3-8 that both the nonlinear frequency and the frequency ratio of microbeams increase as the initial amplitude (α) increases. In these bellow Figures, the Poisson's ratio is chosen as $\nu = 0.3$ for steel material.

Table 1. Comparison of the approximate frequencies with the exact frequency

α	H	P–P microbeam		C–C microbeam	
		$\omega_{present}$	ω_{MSLP} [21]	$\omega_{present}$	ω_{MSLP} [21]
0.1	5	12.3056	12.3059	24.6885	24.6887
0.5		12.4754	12.4827	24.8019	24.8068
0.9		12.8629	12.8859	25.0644	25.0802
0.1	20	17.3053	17.3055	28.4063	28.4064
0.5		17.4264	17.4317	28.5049	28.5091
0.9		17.7059	17.7226	28.7336	28.7473
0.1	50	24.4041	24.4043	34.6655	34.6657
0.5		24.4902	24.4939	34.7464	34.7499
0.9		24.6898	24.7018	34.9343	34.9456

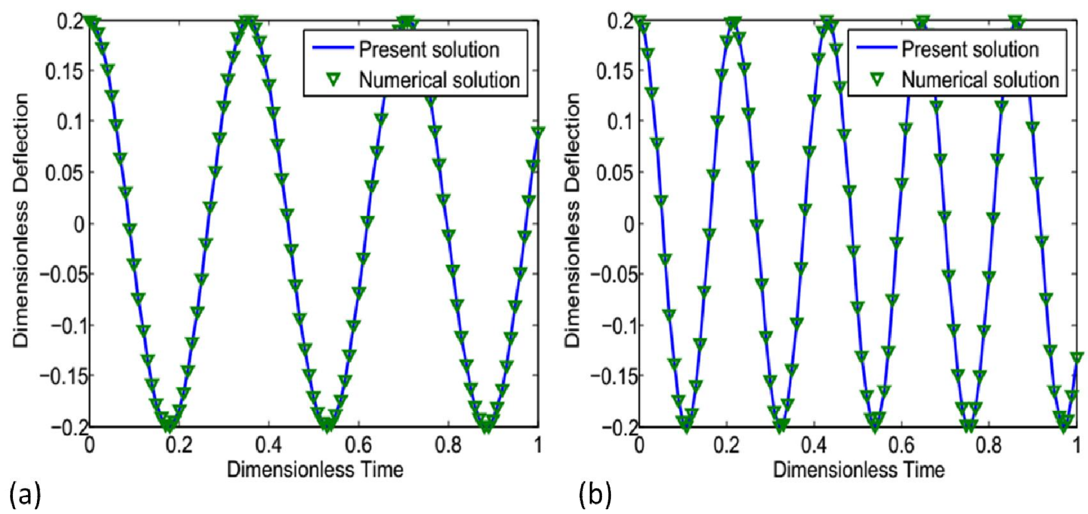

Fig. 2. Time history of responses of P-P microbeam (a) and C-C microbeam (b) for $\nu = 0.3$, $l/h = 0.2$, $H = 10$ and $K_L = 100$

Effects of the material length scale parameter *l/h* and the magnetic field *H* on response of microbeam can be seen from Figs. 3-6. Figs. 3 and 4 plot variations of the nonlinear frequency ω_{NL} and the frequency ratio ω_{NL}/ω_L of microbeam to the initial amplitude α with some different values of the material length scale parameter (*l/h*=0; 0.2; 0.4 and 0.6). And, variations of the nonlinear frequency ω_{NL} and the frequency ratio ω_{NL}/ω_L of microbeam to the initial amplitude α with some different values of the magnetic field (*H*=0; 10; 20 and 40) are showed in Figs. 5 and 6. We can see that the nonlinear frequency ω_{NL} of microbeam increases when the material length scale parameter *l/h* and the magnetic field *H* increase; but on the other hand the frequency ratio ω_{NL}/ω_L of microbeam decreases as the material length scale parameter *l/h* and the magnetic field *H* increase. When increasing invalues of the material length scale parameter *l/h* and the magnetic field *H*, the linear frequency ω_L increases faster than the nonlinear frequency ω_{NL}, which leads to a reduction in the frequency ratio ω_{NL}/ω_L of microbeam.

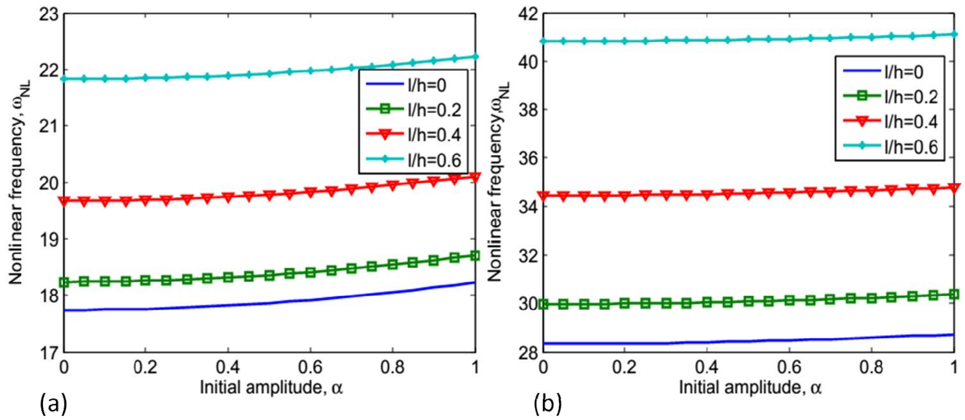

Fig. 3. Variations of the nonlinear frequencies (ω_{NL}) of microbeams to the initial amplitude (α) for *H*=20 and K_L=20; (a) P-P microbeam, (b) C-C microbeam

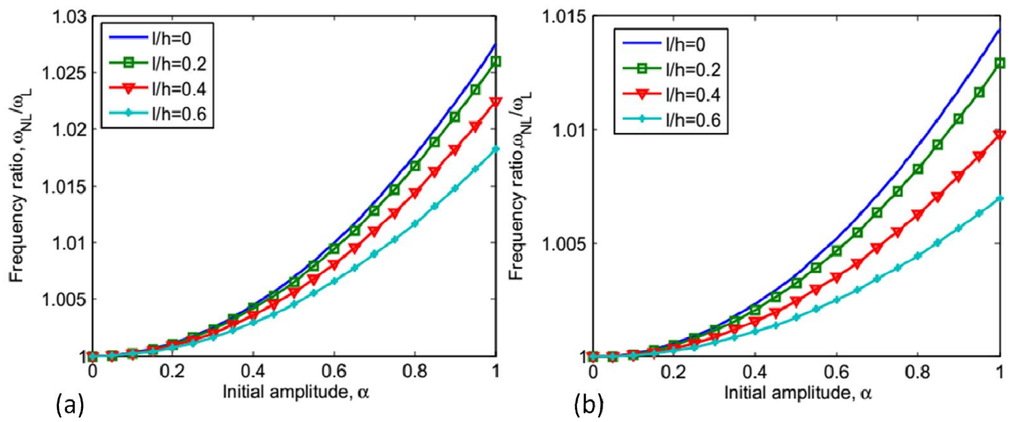

Fig. 4. Variations of the frequency ratios (ω_{NL}/ω_L) of microbeams to the initial amplitude (α) for *H*=20 and K_L=20; (a) P-P microbeam, (b) C-C microbeam

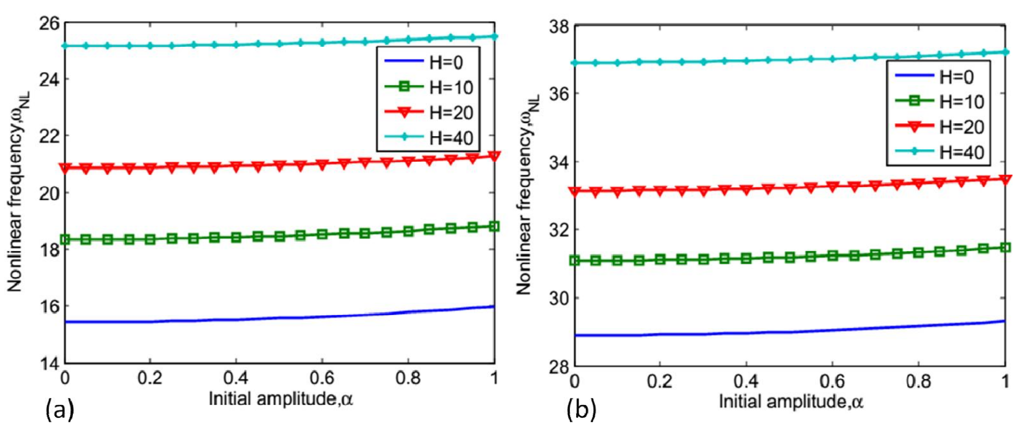

Fig. 5. Variations of the nonlinear frequencies (ω_{NL}) of microbeams to the initial amplitude (α) for *l/h=0.3* and K_L=100; (a) P-Pmicrobeam, (b) C-C microbeam

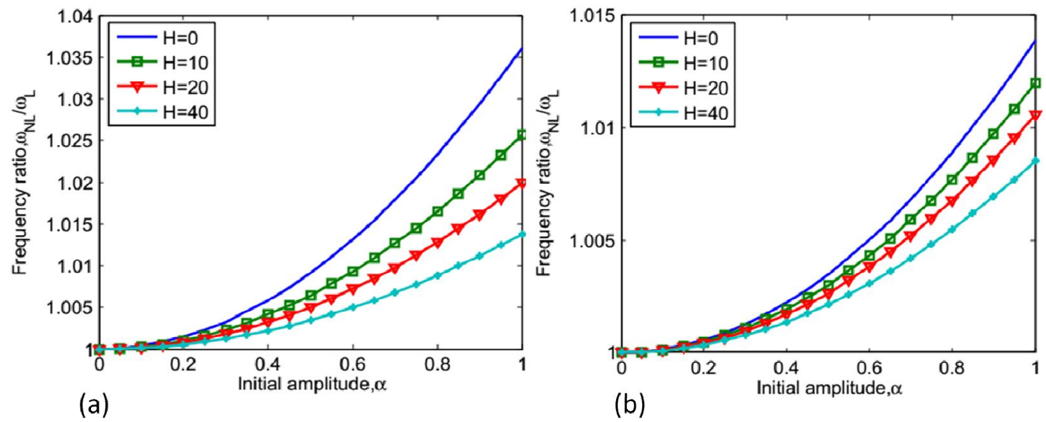

Fig. 6. Variations of the frequency ratios (ω_{NL}/ω_L) of microbeams to the initial amplitude (α) for *l/h=0.3* and K_L=100; (a) P-P microbeam, (b) C-C microbeam

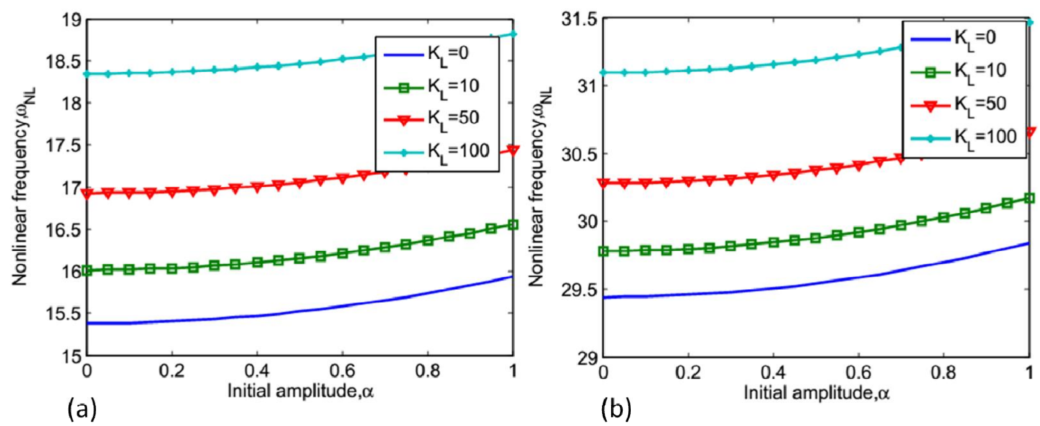

Fig. 7. Variations of the nonlinear frequencies (ω_{NL}) of microbeamsto the initial amplitude (α) for *l/h=0.3* and *H*=10; (a) P-P microbeam, (b) C-C microbeam

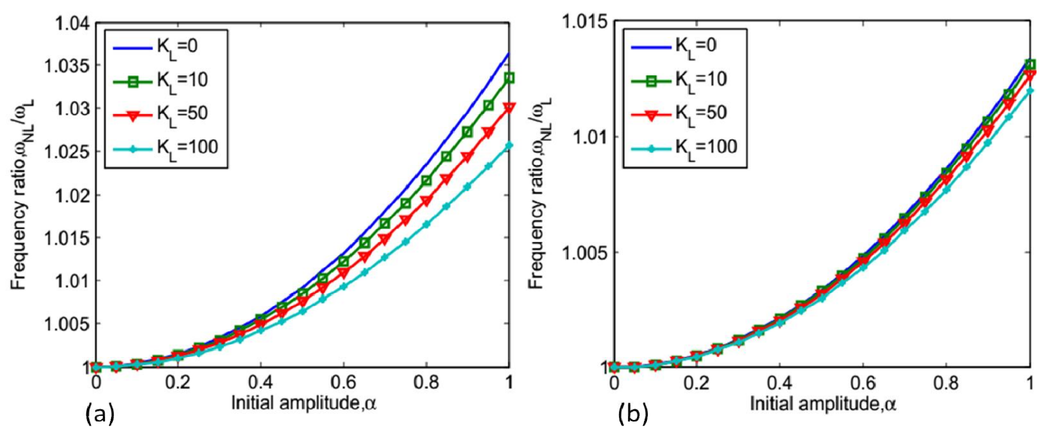

Fig. 8. Variations of the frequency ratios (ω_{NL}/ω_L) of microbeams to the initial amplitude (α) for *l/h=0.3* and *H*=10; (a) P-P microbeam, (b) C-C microbeam

Finally, effect of the Winkler parameter (K_L) of the elastic layer on the nonlinear frequency ω_{NL} and the frequency ratio ω_{NL} / ω_L of microbeam is studied. Variations of the nonlinear frequency ω_{NL} and the frequency ratio ω_{NL} / ω_L to the initial amplitude α for some different values of the Winkler parameter (K_L=0; 10; 50 and 100) are presented in Figs. 7 and 8. It is reasonable that when value of the Winkler parameter (K_L) increases, microbeams become harder and therefore the nonlinear frequencies of microbeams increase, this behavior is called the hardening spring behavior. On the other hand, when the Winkler parameter (K_L) increases, the frequency ratios (ω_{NL} / ω_L) of microbeams decrease.

5 Conclusions

Based on MCST and Euler-Bernoulli beam theory, nonlinear free vibration behavior of microbeam under longitudinal magnetic field is investigated in this work. The governing equation of motion of microbeam is derived by using Hamilton's principle with the Von-Kármán's assumptions about strain-displacement relationships. ELM with a weighted averaging is employed to get the approximate frequencies of microbeams with P-P and C-C end conditions. Comparing the obtained solution with the published solution and the numerical solution shows acurracy of the present solution. Effects of the material length scale parameter *l/h*, the magnetic field *H* and the Winkler parameter K_L of the elastic layer on the nonlinear frequencies ω_{NL} and the frequency ratios ω_{NL} / ω_L of P-P and C-C microbeams are investigated and discussed in the section 4. The nonlinear frequencies of microbeams increase as increasing in values of the material length scale parameter *l/h*, the magnetic field *H* and the Winkler parameter K_L. And on the other hand, when the material length scale parameter *l/h*, the magnetic field *H* and the Winkler parameter K_L increase, the frequency ratios of microbeams decrease.

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Competing Interests

Authors have declared that no competing interests exist.

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