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Exact Solutions with Bounded Periodic Amplitude for Kundu Equation and Derivative Nonlinear Schrödinger Equation

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

In this paper, exact solutions with bounded periodic amplitude to Kundu equation are obtained through transformation, direct integration method and trial function method when the parameters satisfy certain conditions. By the way, exact solutions for the derivative nonlinear Schrödinger equation are also obtained. Two solutions' images are displayed. These results greatly enrich the solutions' structural diversity for these equations.

Keywords: Kundu equation; derivative nonlinear Schrödinger equation; transformation and direct integration method; trial function method; exact solution.

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1 Introduction

Nonlinear Schrödinger type equations have been widely applied to the field of physics, such as plasma physics, nonlinear fluid mechanics, nonlinear optics and quantum physics. In recent decades, this type of equations have attracted many researchers to study, who have gotten fruitful results [1]-[10]. Anjan Kundu derived the following higher-order nonlinear equation (Kundu equation) from nonlinear Schrödinger type equations [11]

$$iu_t + u_{xx} + \beta|u|^2u + \gamma|u|^4u + i\alpha(|u|^2u)_x + is(|u|^2)_xu = 0, \tag{1.1}$$

Kundu equation can be denoted into the following equivalent form

$$iu_t + u_{xx} + \beta|u|^2u + \gamma|u|^4u + i(2\alpha + s)|u|^2u_x + i(\alpha + s)u^2\bar{u}_x = 0. \tag{1.2}$$

When $s = 0$, Eq.(1.1) becomes the derivative nonlinear Schrödinger equation as follows

$$iu_t + u_{xx} + \beta|u|^2u + \gamma|u|^4u + i\alpha(|u|^2u)_x = 0. \tag{1.3}$$

When $\alpha = 0$, $\beta = 2$, $\gamma = 4\delta^2$ and $s = -4\delta$, Eq.(1.1) becomes Kundu-Eckhaus equation

$$iu_t + u_{xx} + 2|u|^2u + 4\delta^2|u|^4u - 4i\delta(|u|^2)_xu = 0,\ \delta \in R, \tag{1.4}$$

In addition, Chen-Lee-Lin equation $iu_t + u_{xx} + i\delta^2|u|^2u_x = 0$ and Gerdjikov-Ivanov equation $iu_t + u_{xx} + \beta|u|^2u + 2\delta^2|u|^4u + 2i\delta u^2\bar{u}_x = 0$ are also special cases of Eq.(1.2).

Many scholars have obtained exact solitary wave solutions, travelling wave solutions and singular periodic solutions for Eq.(1.1) [12]-[15]. Some scholars have also investigated Kundu-Eckhaus equation from which rogue-wave solutions are obtained [16] and [17]. However, to our knowledge, exact solutions with bounded periodic amplitude for Eq.(1.1) have not been reported. Difficulty to look for exact solutions lies in the presence of the fifth-order nonlinear term in this type equations. Recently, we have found that Eq.(1.1) possesses exact solutions with bounded periodic amplitude under a special condition, that is $4\,s^2 - 16\gamma + 4\,\alpha\,s - 3\,\alpha^2 = 0$. The aim of the present paper will be to investigate Eq.(1.1) through transformation, direct integration method and trial function method to obtain exact solutions including trigonometric and elliptic function solutions.

The rest of the paper is organized as follows: In Sect.2, we will simplify the structure of Eq.(1.1) by transformation, and solve simplified equation. Sect.3 will be our conclusions.

2 Exact Solutions with Bounded Periodic Amplitude

By using a transformation

$$u(x, t) = Q(x, t)e^{(iW(x,t)+ivt)}, \tag{2.1}$$

Eq.(1.1) can be transformed into the following equation

$$\begin{aligned}&((3\,\alpha + 2\,s)Q^2(x, t)Q_x(x, t) + Q(x, t)W_{xx}(x, t) + Q_t(x, t) + 2Q_x(x, t)W_x(x, t))i + \gamma Q^5(x, t)\\&+(\beta - \alpha\,W_x(x, t))Q^3(x, t) - (v + W_x^2(x, t) + W_t(x, t))Q(x, t) + Q_{xx}(x, t) = 0.\end{aligned} \tag{2.2}$$

where $Q(x, t)$ and $W(x, t)$ are real functions to be determined, and v is real constant to be determined.

Case 1: If set

$$W_x(x, t) = A + B\,Q^2(x, t),\ W_t(x, t) = E + F\,Q^2(x, t), \tag{2.3}$$

then Eq.(2.2) becomes the following form

$$((3\alpha+2s+4B)Q^2(x,t)Q_x(x,t)+2AQ_x(x,t)+Q_t(x,t))i+Q_{xx}(x,t)-(v+A^2+E)Q(x,t)$$
$$-(A\,\alpha+2\,A\,B+F-\beta)Q^3(x,t)+(\gamma-\alpha\,B-B^2)Q^5(x,t)=0, \tag{2.4}$$

where A, B, E and F are real constants to be determined. When the parameters satisfy the following conditions

$$3\alpha+2s+4B=0,\ A\,\alpha+2\,A\,B+F-\beta=0,\ \gamma-\alpha\,B-B^2=0, \tag{2.5}$$

or

$$A=2\frac{F-\beta}{\alpha+2s},\ B=-\frac{1}{2}s-\frac{3}{4}\alpha, \gamma=\frac{1}{4}s\alpha-\frac{3}{16}\alpha^2+\frac{1}{4}s^2, \tag{2.6}$$

then, Eq.(2.4) is simplified to the following form

$$(2AQ_x(x,t)+Q_t(x,t))i+Q_{xx}(x,t)-(v+A^2+E)Q(x,t)=0. \tag{2.7}$$

Solving equations $2AQ_x(x,t)+Q_t(x,t)=0$, $Q_{xx}(x,t)-(v+A^2+E)Q(x,t)=0$, we have

$$Q(x,t)=C_1\,sin(\sqrt{-v-A^2-E}(x-2At))+C_2\,cos(\sqrt{-v-A^2-E}(x-2At)), \tag{2.8}$$

where C_1, C_2, E and v are arbitrary constants, they should satisfy the condition $-v-A^2-E>0$.

Substituting Eq.(2.8) into Eq.(2.3) and integralling it , we obtain

$$W(x,t)=-\frac{1}{4\sqrt{-v-A^2-E}}(B(C_1^2-C_2^2)\,sin(2\sqrt{-v-A^2-E}(x-2At))$$
$$+2BC_1C_2\,cos(2\sqrt{-v-A^2-E}(x-2At))+\sqrt{-v-A^2-E}(4B(C_1^2+C_2^2)(-\frac{1}{2}x+At)$$
$$-4Et-4Ax-4C_3)+2BC_1C_2). \tag{2.9}$$

By using $W_{xt}(x,t)=W_{tx}(x,t)$, we obtain $F=-2AB$. Therefore, when A, B, α, β, s and γ satisfy the relationships

$$A=\frac{\beta}{\alpha},\ B=-\frac{1}{4}(2s+3\alpha), \gamma=\frac{1}{4}s\alpha-\frac{3}{16}\alpha^2+\frac{1}{4}s^2, \tag{2.10}$$

exact solution of Eq.(1.1) is expressed as

$$u(x,t)=(C_1\,sin(\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}(\frac{\alpha x-2\beta t}{\alpha}))$$
$$+C_2\,cos(\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}(\frac{\alpha x-2\beta t}{\alpha})))e^{(iW(x,t)+ivt)}, \tag{2.11}$$

where C_1, C_2, E and v are arbitrary constants, and

$$W(x,t)=\frac{(2s+3\alpha)}{16\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}}((C_1^2-C_2^2)\,sin(\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}(\frac{\alpha x-2\beta t}{\alpha}))$$
$$+2C_1C_2\,cos(\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}(\frac{\alpha x-2\beta t}{\alpha})))+\frac{C_1C_2(2s+3\alpha)}{8\sqrt{-\frac{v\alpha^2+\beta^2+E\alpha^2}{\alpha^2}}}$$
$$-\frac{1}{8\alpha}(C_1^2+C_2^2)(2s+3\alpha)(\alpha x-2\beta t)+Et+\frac{\beta}{\alpha}x+C_3, \tag{2.12}$$

α, β, v and E satisfy $v\alpha^2+\beta^2+E\alpha^2<0$. Eg.(2.11) is a bounded periodic amplitude solution of Kundu equation (see Fig. 1).

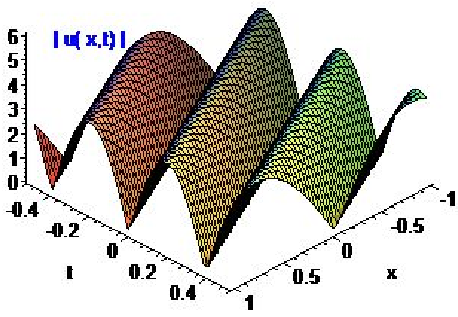

Fig. 1. Profile of $|u(x,t)|$ in Eq.(2.11) with $\alpha = 1$, $\beta = 2$, $v = 2$, $E = -10$, $C_1 = 2$, $C_2 = 3$

Case 2: If set $X = P(x - k\,t)$ and $W_X(X) = A + B\,Q(X)^2$, then Eq.(2.2) can be converted to the following from

$$i((3\alpha + 2s + 4PB)Q(X)^2 + (2PA - k))Q_X(X) + P^2\,Q_{XX}(X) + (-v + P^2\,A^2)Q(X) \\ +(\beta - \alpha\,P\,A)Q(X)^3 + (-\alpha\,P\,B + \gamma - P^2\,B^2)Q(X)^5 = 0. \tag{2.13}$$

When the parameters satisfy the following conditions

$$k = 2PA,\ B = -\tfrac{1}{4P}(2s + 3\alpha), \gamma = \tfrac{1}{4}s\alpha - \tfrac{3}{16}\alpha^2 + \tfrac{1}{4}s^2, \tag{2.14}$$

Eq.(2.13) is simplified to the following form

$$P^2\,Q_{XX}(X) + (-v + P^2\,A^2)Q(X) + (\beta - \alpha\,P\,A)Q(X)^3 = 0. \tag{2.15}$$

We use trial function method to look for elliptic function solutions for Eq.(2.15). Suppose solution of Eq.(2.15) as follows

$$Q(X) = a_0 + a_1\,JacobiSN(X, m), \tag{2.16}$$

where a_0, a_1, b_1 and $m\,(0 < m < 1)$ are constants to be determined. Substituting Eq.(2.16) into Eq.(2.15), we easily obtain the following results.

When $a_0 = 0$, $A = \frac{2\,P^2m^2+\beta\,a_1^2}{\alpha P\,a_1^2}$, $v = -\frac{P^2m^2a^2a_1^4-4P^4m^4-4\beta P^2m^2a_1^2-\beta^2a_1^4+\alpha^2P^2a_1^4}{\alpha^2a_1^4}$, Eq.(2.15) has solution

$$Q(X) = a_1\,JacobiSN(X, m), \tag{2.17}$$

where a_1, P and $m\,(0 < m < 1)$ are arbitrary constants. At this time, the elliptic function solution of Eq.(1.1) is expressed as

$$u(x,t) = a_1\,JacobiSN(X,m)\,e^{(i\int(\frac{2\,P^2m^2+\beta\,b_1^2}{\alpha P\,b_1^2} - \frac{1}{4P}(2s+3\alpha)(a_1\,JacobiSN(X,m))^2)dX + ivt)}, \tag{2.18}$$

where $X = P(x - 2PAt)$ and $v = -\frac{P^2m^2a^2a_1^4-4P^4m^4-4\beta P^2m^2a_1^2-\beta^2a_1^4+\alpha^2P^2a_1^4}{\alpha^2a_1^4}$. This is a bounded elliptic function solution of Kundu equation (see Fig. 2).

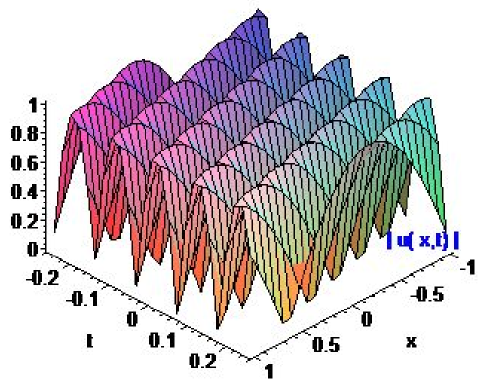

Fig. 2. Profile of $|u(x,t)|$ in Eq.(2.18) with $\alpha = \frac{1}{2}$, $\beta = 2$, $m = \frac{1}{2}$, $a_1 = 1$, $P = 2$

In the above solutions, setting $s = 0$, we can obtain solutions of the derivative nonlinear Schrödinger equation.

3 Conclusion

When parameters satisfy condition $4\,s^2 - 16\gamma + 4\,\alpha\,s - 3\,\alpha^2 = 0$ in Kundu equation, its bounded periodic amplitude solutions including trigonometric and elliptic function solutions, are obtained. Prior to this, bounded periodic amplitude solutions have not reported. Based on the solutions of Kundu equation, we easily obtain solutions to the derivative nonlinear Schrödinger equation. These results contribute to a better understanding of the structure of the solutions for the nonlinear Schrödinger type equations. They can also be applied to the field of nonlinear optics.

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Competing Interests

Authors have declared that no competing interests exist.

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