



The Nature of Mathematics Education; The Issue of Learning Theories and Classroom Practice

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ABSTRACT

To achieve a successful mathematics education through the implementation of the school mathematics curriculum, the teacher's beliefs emanating from their experiences (including learning theories) with their environment should be a considerable factor. This is to ensure that all learning situation in mathematics classroom generate understanding among the individuals involve for it to be meaningful. In this paper my goal is to highlight on some issues which are of great importance to mathematics education as a field of study. The issues in focus are; theory and practice, the role of theory in mathematics education research, some widely used learning theories in mathematics education, theories of models and modelling and their impact on mathematics research, teaching and learning, the aims of teaching and learning mathematics, and the status of mathematics education as knowledge field.

Keywords: Theories; mathematics education; teaching; learning.

1. INTRODUCTION

The backbone of every successful educational process depends on the teacher to a large

extent. Teaching is a perceived system of actions taken to induce learning through interpersonal interactions. A study by [1] suggested that "The good teaching is explaining, informing, initiating,

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directing, administering, unifying the group, giving security, clarifying, diagnosing, learning problems, preparing curriculum material, evaluating, recording, reporting, enriching community activities, organizing and arranging classrooms, participating in school activities and in professional development". This suggests that mathematics education provides people with an opportunity to reflect on the critical, social, economic, cultural, moral and spiritual issues facing humanity. It therefore contributes to national development through dissemination of specialized knowledge and skills which is considered as a crucial factor for survival.

The main aim of mathematics education is to effect learning and understanding. In the view of [2], they posited that "Learning is the acquisition of habits, knowledge, and attitude. It involves new ways of doing things and it operates on individual attempts to overcome obstacles or to adjust to new situations. It represents progressive change in behavior. It enables him to satisfy interests to attain goals". A good teaching and learning lead to good understanding. Research by [3] proposed probably the most comprehensive work combining different aspects related to understanding. They came up with definitions for five strands of mathematical proficiency. These are:

- Conceptual understanding – comprehension of mathematical concepts, operations, and relations.
- Procedural fluency – skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.
- Strategic knowledge – ability to formulate, represent, and solve mathematics problems.
- Adaptive reasoning – capacity for logical thought, reflection, explanation, and justification.
- Productive disposition – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (p. 116).

They [10] further stated that these suggested strands are interwoven and interdependent in the development of an individual's mathematical proficiency.

Some researchers also advocated that understanding is not only mental by mentioning the social aspects of understanding [4,5]. Some

researchers have suggested that understanding helps students use their knowledge in different situations [6,7,5], making learning with understanding the greatest benefit to students. [6] mentioned that understanding can help one to remember things more easily; and even that there is less to remember if someone understands a mathematical idea. It is believed that conceptually grounded ideas are more accessible in new situations because connections facilitate the transfer of knowledge [7]. [8] found that understanding can help to reconstruct certain procedures, since rich internal links among mathematical ideas may not be easily forgotten because they enhance memory. Learning with understanding lead to the idea that mathematics is fun [9]. In a similarly fashion, [8] mentioned that understanding is intrinsically rewarding and that it improves attitudes and beliefs regarding mathematics. [9] further explained that understanding can provide students with the idea that mathematics is useful. It is believe that learning with understanding is self-generative [8].

This means that learning can create a snowball effect so that understanding something can lead to learning new things. It suggests that the belief of a teacher which are influenced by theories can affect how they respond to proposals for changes in instructional practice and the way they teach in their classrooms.

2. THEORY AND PRACTICE

The concept of theory can be described as the group of predictions that put forward proposals to find the reasons why events occur. Theory is, in a way, a strategy that helps to realize those ideas in line with the plans previously drawn up. Simultaneously, theory is a path that is taken as the basis for moving, and is followed accordingly during teaching [10]. The bare fact of the issue is, all practice in education, including mathematics education and as well as other fields rely on theories. This suggests that, as teachers, our classroom behavior and statements are mostly guided by theories that we come across during our training.

Therefore, doing teaching requires a solid understanding of the basic theories that drive teaching, including ideas about how students learn, what they should learn, and how teachers can make learning possible for students in our classrooms. Additionally, one important idea about what counts as knowledge and what

students can learn is that students need to gain broad understanding, including both essential factual and conceptual knowledge, and need to know how to make practical use of that knowledge is not left behind in teaching and learning. We may conclude that teaching is intellectual work; teachers have a variety of roles, including knowledge providers and team coaches; successful teachers strategically communicate (or share) work with students; and teachers concentrate on challenging content.

Research suggests that one of the main source from which we develop our learning theories that form the foundation of our attitudes and behavior in education is personal experience [11]. As teachers continually interact with students in the school environment through all manner of ways, they in return develop so much experience over time. These experiences can lead the teacher into an adventurous situations like;

- Modifying an existing theory; or
- Proposing a new theory where possible; or
- Doing further research to fine-tune an existing theory.

3. THE ROLE OF THEORY IN MATHEMATICS EDUCATION RESEARCH

A theory is to provide instructions for action or actions by using knowledge of claims and the conditions of their validity in theory to plan and enforce action or behavior to obtain beneficial or to avoid undesirable results. It also provides a protection against unscientific approaches to a topic, issue or theme, including, for example, haphazard and incoherent terminology choices, research methods, and results interpretation. This aim is achieved by articulating underlying theories and choices and by making them clear and open to discussion; by putting one's study within a certain framework; and by announcing and explaining its characteristics with respect to potential alternatives.

In addition, the intent of a theory is to provide a standardized collection of lenses through which to approach, observe, research, evaluate or interpret aspects or parts of the world. This is achieved by choosing the elements that are relevant to include in the context, while omitting others; by concentrating on certain features or issues; by implementing and using specific perspectives; and by providing the entire community with a methodology. This offers

defense against attacks by cynical or aggressive colleagues in other areas of study. For example, researchers in mathematics education often experience criticism of our field from outside colleagues including those in pure mathematics, psychology, or general education and so on regarding the basis of our work and its outcome. Once theory is used as a guide, it may help to:

- Envision ways of enhancing the teaching/learning environment, including the curriculum;
- Establish methodology;
- Characterize, view, clarify and justify student and teacher findings in the classroom;
- Turn practical problems into research problems;
- Identify various steps in the study of a research problem; and
- Create research problems.

However, when theory acts as an object or entity, the development of the theory itself will be one of its objectives. This may involve evaluating a theory or other theory ideas or relationships in a particular context, or as a means of generating new theoretical innovations.

4. SOME WIDELY USED LEARNING THEORIES IN MATHEMATICS EDUCATION

Such body of knowledge contribute to the teaching of mathematical concepts that ensure the connection between teacher and learner, and any learning-based comprehension through the active involvement of students through the use of their information and experience in creating new experience that effectively leads them into new situations. The generally accepted and commonly used theories of learning in math education include, but not limited to; Formalism, structuralism, empiricism, radical constructivism, social constructivism, semiotics, theories of (applied) statistics, behaviourism and neo-behaviourism, cognitive structuralism, cognitive science in general, activity theory, psychoanalysis, APOS theory (Action-Process-Object-Scheme), concept definition and concept image theory, didactical situations (Brousseau's theory), didactical transposition (Chevallard's theory), conceptual fields (Vergnaud's theory), socio-mathematical norms, mathematics teaching theory, teachers' mathematical knowledge for teaching, and critical mathematical education.

However, most of the mathematics curriculums across the world encourages and recommend the use of renowned theories of learning in mathematics education such as constructivism. Constructivism learning theory: is a method of teaching mathematical concepts that ensures engagement between teacher and learner, any learning-based understanding by students' active involvement through the use of their information and experience in creating new knowledge that leads them into new situations and this is achieved in four successive stages. These are invitations, explorations, suggesting explanations and solutions, and taking decisions. This theory influence the teaching and learning of mathematics through;

- Inviting students to learn through a variety of ways. Teacher to ask them some questions with thought-provoking questions to be borne in mind that at this stage their knowledge related to the previous learning.
- Challenging the capabilities of the students. This stage involves the search for answers to questions specific to their generated learning through observation, measurement and experimentation, and working groups, with each group and each specific tasks of their own.
- Allowing all the groups to provide their findings and interpretations of solutions and proposals and discussed, and where students make mistakes they modify their misconceptions or bring scientific concepts by replacing misconceptions.
- Guide learners where possible to make practical applications of the findings of their solutions and conclusions [12].

When new horizons in mathematics education research are raised, these theories gained attention, we look to other fields that have dealt with similar issues and topics and seek their guidance. Because of the above explanation, theories beyond psychology and statistics were implemented more or less one by one in mathematics education research, along with the introduction of new research topics and focuses on the field agenda. Such concerns and focuses include curriculum reform; application of mathematics to extra-mathematical fields; philosophical features of mathematics; mathematics classrooms; gender issues; linguistic issues; socio-cultural issues, including minority issues; student beliefs, affections, job perspectives; teacher education, in-service

training, and attitudes and beliefs of teachers [13].

However, as it turns out that the assistance offered by other disciplines is somewhat limited because they cannot compensate for the important role of mathematics in the teaching and learning of mathematics, it is necessary to change, recreate or combine existing theories in order to adapt them to the needs of mathematics education and eventually to develop new theories. This actions will contribute to several new emerging ideas or theories in the area of mathematics education as a research field.

5. THEORIES OF MODELS AND MODELLING AND THEIR IMPACT ON MATHEMATICS RESEARCH, TEACHING AND LEARNING

Mathematical modeling, such as teaching technique and learning, starts with a theme/subject, and then questions about it arise. These questions can be answered with the use of mathematical tools and the established thematic work. To an extent, it is highly satisfying to study a subject that can help students obtain a substantial amount of mathematical knowledge, or to acquire information on the theme being studied. In modelling education, each student can choose a theme/subject in any field in his/her interest, to do research on it, formulate questions and with the teacher's guidance, develop a mathematical model. Students thus are co-responsible for their learning, and the teacher becomes a counselor. Learning is richer given that the student not only studies mathematics introduced in the form of another field of knowledge, but has also stirred up his analytical and inventive senses.

Modeling education is driven by the teaching of the content of the curriculum, starting with applied mathematical models and then progressing into many areas of knowledge; at the same time directing students to an enquiry study. It is a method of teaching mathematics which aims to give the student a better understanding of mathematical concepts; to prepare him / her to read, comprehend, formulate and solve specific situation-problems, and to awaken his/her critical and inventive senses. It can be implemented from elementary school teaching through to the tertiary level. Research has shown that the implementation of the approach is supposed to provide the student with:

- integration of mathematics with other areas of knowledge;
- interest in the application of mathematics;
- improvement in grasping mathematical concepts;
- incentive for creativity in the formulation and resolution of problems;
- ability to use of machines (graphic calculators and computers);
- capacity to act in a group;
- orientation in doing research and capacity for reporting research [14].

To implement it in teaching, the instructor chooses a theme/subject of any field of knowledge that may be of interest to students (depending on the content of the program), and develops a mathematical model, modifying it to teach. Alternatively, he/she opts for an established mathematical model and adapts it to program content creation. Therefore, the model would act as a guideline or template. It requires the instructor in a series of activities/stages to provide clear understanding of the topic being discussed.

6. THE AIMS OF TEACHING AND LEARNING MATHEMATICS

Mathematical study itself, as well as intellectual life, has occupied an integral part of human civilization. As a matter of fact, mathematics has been taught as the core subject in the history of school curriculum everywhere in any civilized part of the world [15]. National Policy on Education [16] highlighted, "Mathematics should be visualized as the vehicle to train a child to think, reason, analyse, and articulate logically. Apart from being a specific subject it should be

treated as a concomitant to any subject involving analysis and meaning". Its system of knowledge, together with its language and method, has been communicated among the world intellectual communities more than any other discipline, perhaps more than any other human undertaking.

There may be four explanations as to why we should teach mathematics at school, from the viewpoint of societal benefit. These are;

- to boost mathematical specialists;
- to meet the needs of mathematical knowledge needed for the advanced level of professional services;
- to promote problem-solving skills, including those of logical or formal reasoning; and,
- to help people familiarize themselves with the basic mathematical knowledge necessary for their everyday lives.

Through the teaching of Mathematics, the power of critical thinking, logical reasoning, induction process, and generalization, and the establishment of relationships between different components can be established. To realize the objectives of learning mathematics in general, the following rules are to be observed when teaching mathematics:

- Make mathematics interesting and practical.
- Consider the previous knowledge of the student.
- Make the curriculum for mathematics socially engaging.
- Frame groundbreaking or innovative projects in mathematics.

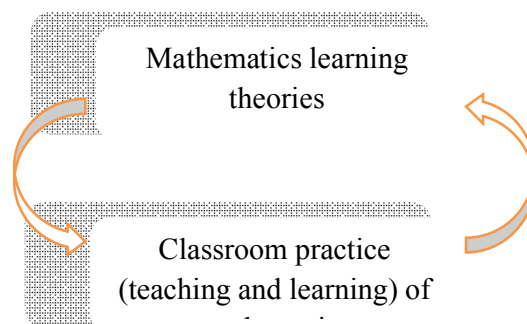


Fig. 1. Conceptual framework showing relation between theories and practice in mathematics education

Today's mathematical form has increasingly new applications for daily life, and the exponential growth of desirable applications is helping to generate more and more new domains of mathematics. This supports the reasons to keep mathematics in the school curriculum since it:

- Serves as foundation of all sciences.
- Corresponds to other subjects and to human life.
- Generates logical attitude.
- Establishes a definite mindset.
- Offers an opportunity to improve the learners' intellectual abilities.

7. THE STATUS OF MATHEMATICS EDUCATION AS KNOWLEDGE FIELD

Recent developments of critical mathematics education issues are very necessary for the future of mathematics. Looking into the future, it looks like much more is on its way, besides a number of research has been done on the future of mathematics education [17,18,19]. Ana Carolina Faustino's study dealt with dialogical mechanisms in primary mathematics education. The dialogic education has evolved, with many parallels to adult education. Faustino, however, did a rework on the very concept of dialogue, with special regard to younger children. Amanda Queiroz Moura explores the education of deaf students in mathematics. It allows her to tackle different aspects of inclusive education as well as dialogical education, thereby providing new dimensions for the discussion of mathematics education for social justice [17].

João Luiz Muzinatti works with middle-class students in another study, and he questions a number of perceptions and preconceptions that govern middle-class discourses through various mathematical projects. Thus Muzinatti formulated new considerations for critical education in mathematics [19]. Affirmative actions was discussed by Guilherme Henrique Gomes da Silva. He examines the various components of such actions, emphasizing that the very educational nature of university studies must also be addressed by affirmative actions. That way da Silva added a new scope to the debate of affirmative actions in teaching and learning mathematics [17].

This suggests that mathematics education has tried to address issues concerning discourse or dialogue among the young and the adult, inclusive education for promoting social justice,

affirmative actions to address all levels of learning and many more as a knowledge field. Additionally, the researchers in mathematics education field are continually addressing mathematics in its very many different forms of applications and practices.

8. SUMMARY AND CONCLUSION

There are various approaches to mathematics teaching and learning base on theories. Broadly speaking these approaches are teacher centered and learner centered. This may require the use of strategies either individually, grouped or cherry picked. These approaches are ways to work through a problem, whether it's a program, project, risk, questions and many more. It is through this that [20] identified seven styles of teaching mathematics to aid effective learning. They are as follows:

1. Child centered style: Here the instructor provides a framework, and the students pursue it in accordance with their desires and likes.
2. Teacher centered style: In this, the teacher plays a major role. The teaching learning process is dictated by the teacher. Learners are passive listeners.
3. Subject Centered style: Here the teacher's main emphasis is on the structured content of the subject matter that should be presented to the learners.
4. Task Centered Style: Here learning is seen on individual bases as the teacher prescribes the material to be learnt by the learners and then he expects specific performance on the part of the students.
5. Learning Centered Style: Opposite of child centered and subject centered styles, the focus here is on the learners and their learning regardless of their abilities and handicaps.
6. Co-operative Planner Style: Means and ends of instructions are planned by the teacher in cooperation with the learners. Here the teacher inspires and assists the learners in every way.
7. Emotional Involvement Style: The teacher is affectionately involved in this style of teaching. He enters the teaching learning process with zeal and dedication and thus he succeeds in producing good class room environment.

As we know, the main aim of education is to create the necessary improvements in the

children's behavior, and when those improvements have happened, we conclude the child has learned. Mathematics teachers are encouraged to adopt learning theories that maximize teaching and learning and bring about effective understanding of mathematical concepts in their classrooms.

I believe that as experienced teachers that we are, all teachers operate according to theories. Usually, our practice is driven by our theories regarding what is going to work for our students. Although many people would like to argue that teachers are born and not created, I believe that good teaching is realized when teachers create and utilize, modify or reject, construct or reconstruct learning and teaching theories to aid their practice. Those theories, however, are not based on intuitions, but rather, they are carefully crafted lessons learned from years of experience and careful investigation or research. To become accomplished practitioners, we therefore, need to be interrogating our own theories and assumptions that are produced by our new generations of education scholars in moving forward.

DISCLAIMER

The products used for this research are commonly and predominantly used products in our area of research and country. There is absolutely no conflict of interest between the author and producers of the products because we do not intend to use these products as an avenue for any litigation but for the advancement of knowledge. Also, the research was not funded by the producing company rather it was funded by personal effort of the author.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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