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Vibration Reduction of Two Degree of Freedom Nonlinear System Subject to Parametric Excitation via Negative Feedback Velocity

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Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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Abstract

Negative velocity controller (NVC) is used to reduce the vibration of a two degree of freedom nonlinear system subjected to parametric excitation forces. The vibrating motion of the system described by two coupled differential equation, the worst resonance case of the system near the sub-harmonic resonance ($\omega \simeq 2\omega_2$). The method of multiple scales perturbation technique (MSPT) is applied to obtain the periodic response equation near the selected resonance case. Study the controls on the worst resonance case numerically. The stability of the obtained numerical solution is investigated using both phase plane methods and frequency response equations. Effects of different parameters on the system behavior are studied numerically.

Keywords: Stability; frequency response; multiple times scale; vibration control; parametric excitation.

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1 Introduction

The vibration and dynamical chaos are sometimes undesired phenomenon such that the dynamical response of mechanical and civil structures subject to high amplitude vibration is often disturbance, discomfort, damage, dangerous and destruction of the system or the structure, so the vibration in such systems are needed to be controlled to minimize or eliminate the hazard of damage or destruction . The nonlinear dynamics of two degree of freedom vibration system including quadratic and cubic nonlinearities subjected to external and parametric excitation forces is investigated by Sayed et al. [1,2].

Kamel et al. [3] discussed the system ultrasonic machining using passive control to reduce the vibration in the tool holder and have reasonable amplitude for the tools which it is interest for the machining of non-conductive, brittle materials such as engineering ceramics, a multi-tool technique. Rup-inder [4] and Aspinwall [5] introduced a review for the fundamental principles of stationary ultrasonic machining, the material removal mechanisms involved and the effect of parameters.

Amer [6] investigated the coupling of two nonlinear oscillators of the main system and absorber representing ultrasonic cutting process subjected to parametric excitation forces, he obtained a threshold value of the main system linear damping, where vibration can be reduced dramatically. Many nonlinear dynamical systems using the passive vibration control to reduce the vibration are studied in Asfer [7], Eissa and Abdelhafez [8], EL-Bassiouny [9,10] and Shitikova [11]. Eissa et al. [12-14], Amer and Bauomy [15], and Jaensch [16] presented how the active control is effective in vibration reduction at the resonance at different models of vibration. Eissa and EL-Ganaini [17,18] studied the control of both vibration and dynamics chaos of mechanical system having quadratic and cubic nonlinearities, subjected to harmonic excitation using multi-absorbers. Hamed [19] studied of an application of magnetorheological and semi-active control to isolate the vibration of autoparametric system composed of a nonlinear oscillator with an attached pendulum. Abdelhafez and Nassar [20] studied quantitative analysis on the nonlinear behavior of a forced and self-excited beam coupled with a positive position feedback controller PPF. Such that the external excitation is a harmonic motion on the support of the cantilever beam. Kecik and Borowiec [21] presented a numerical study of an autoparametric system composed a pendulum and an excited nonlinear oscillator. Kecik and Warminski [22] showed the chaotic motion in instability region, chaotic swings and chaotic motion composed of swings and rotation of pendulum are discussed.

Hamed et al. [23-25] investigated the effects of an active vibration control on a nonlinear two-degree-of-freedom system described by a nonlinear differential equations subjected to mixed excitation forces, the stability of the systems is investigated with frequency response curves and phase-plane method. Yuejing Zhao et al. [26] conducted the configuration and force analysis of vertical vibratory conveyor. The model of system with considering the friction between the materials and the spiral conveying trough is developed. The numerical simulations are done and the dynamical responses curves are studied.

Bayıroglu [27] studied the primary, Subharmonic, and superharmonic responses along with numerical methods for vertical conveyors. The change in the parameters of motion, stability condition, and jump phenomena has been shown graphically by mathematical software for comparing the results. Bayıroğlu [28] analyzed the nonlinear analysis of unbalanced mass of vertical conveyor with non-ideal DC motor. Sayed et al. [29] obtained the analytical and numerical study to investigate the vibration and stability of the Van der Pol equation subjected to external and parametric excitation forces via feedback control. El-Sayed and Bauomy [30] used the two positive position feedback controllers (PPF) are used to reduce the vertical vibration in the vertical conveyors. An investigation is presented of the response of a four degree-of-freedom system (4-DOF) with cubic nonlinearities and external excitations at primary resonance. Hamed et al. [31] investigated the nonlinear vibrations and stability of the MEMS gyroscope subjected to different types of parametric excitations using the averaging method to obtain the frequency response equations for the case of sub-harmonic resonance in the presence of 1:1 internal resonances, the stability of the system is investigated with frequency response curves and phase-plane method.

In this article we study effect of two different controls (linear and cubic) feedback on the system and ability of both on absorber to control. The method of multiple time scales is perturbation and the technique is applied to obtain the periodic response equation near the selected resonance case.

2 Mathematical Modeling

The system consists of parametric excited 2dof system of linear coupled oscillators (with identical mass) and nonlinear energy sinks (NES) attached to it. By the term NES we mean a small mass (relative to the linear oscillator mass) attached via essentially nonlinear spring and damping to the linear subsystem [32].

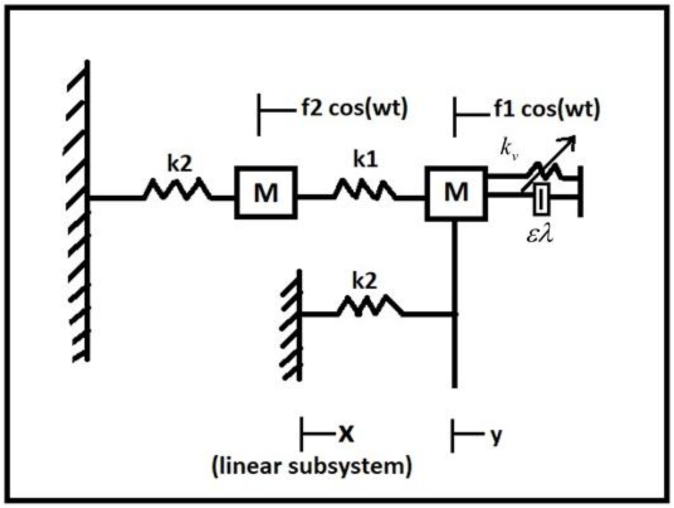


Fig. 1. Mechanical model of the system

2.1 System with Linear control

$$\ddot{y}+\omega_2^2 y+\varepsilon\omega_1^2(y-x)+\varepsilon k_v y^3+\varepsilon\lambda\dot{y}=\varepsilon f_1 y\cos\omega t-\varepsilon v_1\dot{y} \tag{1}$$

$$\ddot{x}+\omega_2^2 x+\varepsilon\omega_1^2(x-y)=\varepsilon f_2 x\cos\omega t-\varepsilon v_2\dot{x} \tag{2}$$

where *y, x* are the displacements of the linear oscillators and nonlinear energy sink (NES), $\ddot{y}$ $\ddot{x}$ derivatives of y, x, ελ is the damping coefficient, εf_i *(i=1, 2)* are the amplitudes of excitation of each linear oscillator, ε is a small parameter, ($\omega_1^2=\dfrac{k_1}{M}, \omega_2^2=\dfrac{k_2}{M}$), are natural frequencies and ω is forcing frequency.

2.1.1 Mathematical analysis

Using the multiple time scale perturbation technique the analytical solutions of equations (1) and (2) are given by:

$$x(T_0;T_1)=x_0(T_0;T_1)+\varepsilon x_1(T_0;T_1)+\ldots\ldots \tag{3}$$

$$y(T_0;T_1) = y_0(T_0;T_1) + \varepsilon y_1(T_0;T_1) + \ldots\ldots \tag{4}$$

where T_0=t is fast time scale, which is associated with changes occurring at the frequencies, ω_1 ω_2 and ω, and T_1= εt is the slow time scale, which is associated with modulations in the amplitudes and phases resulting from the non-linearity's and parametric resonance. In term of T_0 and T_1 the time derivatives became

$$\frac{d}{dt} = D_0 + \varepsilon D_1 + \ldots\ldots \tag{5}$$

$$\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 + \varepsilon^2 D_1^2 + \ldots\ldots \tag{6}$$

where D_n differential operators; $D_n = \frac{\partial}{\partial T_n}$ (n=0, 1). Substituting Equations (3) and (4) into Equations (1) and (2) and equation the coefficients of same power of ε in both sides, we obtain:

Order (ε^0):

$$(D_0^2 + \omega_2^2) y_0 = 0 \tag{7}$$

$$(D_0^2 + \omega_2^2) x_0 = 0 \tag{8}$$

Order (ε^1):

$$(D_0^2 + \omega_2^2) y_1 = -2D_0 D_1 y_0 - \omega_1^2 y_0 + \omega_1^2 x_0 - k_v y_0^3 - \lambda D_0 y_0 + \frac{f_1}{2} y_0 \cos(\omega t) - v_1 D_0 y_0 \tag{9}$$

$$(D_0^2 + \omega_2^2) x_1 = -2D_0 D_1 x_0 - \omega_1^2 x_0 + \omega_1^2 y_0 + \frac{f_2}{2} y_0 \cos(\omega t) - v_2 D_0 y_0 \tag{10}$$

The general solution of Equations (7) and (8) can be expressed in the form:

$$y_0(T_0, T_1) = A_0 e^{i\omega_2 T_0} + cc \tag{11}$$

$$x_0(T_0, T_1) = B_0 e^{i\omega_2 T_0} + cc \tag{12}$$

where A_0 and B_0 are unknown function in T_1, which can be determined by imposing the solvability condition at the next approximation order by eliminating the secular and small- divisor terms.
Substituting Equations (11) and (12) into Equations (9) and (10) we get:

$$(D_0^2+\omega_2^2)y_1=(-2i\omega_2 D_1 A_0-\omega_1^2 A_0+\omega_1^2 B_0-3k_v A_0^2\overline{A}_0-i\omega_2\lambda A_0-i\omega_2 v_1 A_0)e^{i\omega_2 T_0}$$
$$+(2i\omega_2 D_1\overline{A_0}-\omega_1^2\overline{A_0}+\omega_1^2\overline{B_0}-3k_v\overline{A_0^2}+i\omega_2\lambda\overline{A_0}+i\omega_2 v_1\overline{A_0})e^{-i\omega_2 T_0}$$
$$-k_v A_0^3 e^{3i\omega_2 T_0}-k_v\overline{A}_0^3 e^{-3i\omega_2 T_0}+\frac{f_1}{2}(A_0 e^{i(\omega+\omega_2)T_0}+\overline{A}_0 e^{(\omega+\omega_2)T_0})$$
$$+\frac{f_1}{2}(A_0 e^{-i(\omega-\omega_2)T_0}+\overline{A}_0 e^{i(\omega-\omega_2)T_0}) \tag{13}$$

$$(D_0^2+\omega_2^2)x_1=(-2i\omega_2 D_1 B_0-\omega_1^2 B_0+\omega_1^2 A_0-i\omega_2 v_2 B_0)e^{i\omega_2 T_0}+(2i\omega_2 D_1\overline{B_0}-\omega_1^2\overline{B_0}+\omega_1^2\overline{A_0}$$
$$+i\omega_2 v_2\overline{B_0})e^{-i\omega_2 T_0}+\frac{f_2}{2}(B_0 e^{i(\omega+\omega_2)T_0}+\overline{B}_0 e^{-i(\omega+\omega_2)T_0})+\frac{f_2}{2}(B_0 e^{-i(\omega-\omega_2)T_0}$$
$$+\overline{B}_0 e^{i(\omega-\omega_2)T_0}) \tag{14}$$

After eliminating the secular terms, the general solution of equations (13), (14) is given by:

$$y_1(T_0,T_1)=A_1 e^{i\omega_2 T_0}+\xi_1 e^{3i\omega_2 T_0}+\xi_2 e^{i(\omega+\omega_2)T_0}+\xi_3 e^{i(\omega-\omega_2)T_0}+cc \tag{15}$$

$$x_1(T_0,T_1)=B_1 e^{i\omega_2 T_0}+\xi_4 e^{i(\omega+\omega_2)T_0}+\xi_5 e^{i(\omega-\omega_2)T_0}+cc \tag{16}$$

where (ξ_i , i=1…5) and A_1 , B_1 are complex function in T_1 , and cc is complex conjugate of the preceding terms.

2.1.2 Stability analysis

After numerically studying the different resonance cases and deduce the worst ones, one of the worst cases has been chosen to study the system stability. The selected resonance case is the Sub-harmonic case ($\omega=2\omega_2$). In this case we introduce the detuning parameter σ according to:

$$\omega=2\omega_2+\sigma T_1 \tag{17}$$

where σ is the detuning parameter .Also for stability investigation, the analysis is limited to the first approximation . Substituting Eq. (17) into Equations (13) and (14) and eliminating the secular terms leads to the solvability conditions

$$(-2i\omega_2 D_1 A_0-\omega_1^2 A_0+\omega_1^2 B_0-3k_v A_0^2\overline{A}_0-i\omega_2\lambda A_0-i\omega_2 v_1 A_0+\frac{f_1}{2}\overline{A}_0 e^{i\sigma T_1})=0 \tag{18}$$

$$(-2i\omega_2 D_1 B_0-\omega_1^2 B_0+\omega_1^2 A_0+i\omega_2 v_2\overline{B_0}+\frac{f_2}{2}\overline{B}_0 e^{i\sigma T_1})=0 \tag{19}$$

To analyze the solution of Equations (18) and (19), it is convenient to express A_0 in the polar form as:

$$A_0(T_1) = \frac{1}{2} a_1(T_1) e^{i\theta_1(T_1)} \tag{20}$$

$$B_0(T_1) = \frac{1}{2} a_2(T_1) e^{i\theta_2(T_1)} \tag{21}$$

where $a_i(i=1,2), \theta_i(i=1,2)$ are unknown real-valued function .Inserting equation (20), (21) into Equations (18), (19) and separating the real and imaginary parts we have the following:

$$a_1' = -\frac{1}{2}(\lambda + v_1)a_1 + \frac{f_1}{4\omega_2} a_1 \sin(\gamma_1) + \frac{\omega_1^2}{2\omega_2} a_2 \sin(\gamma) \tag{22}$$

$$\theta_1' a_1 = \frac{\omega_1^2}{2\omega_2} a_1 + \frac{3k_v}{8\omega_2} a_1^3 - \frac{f_1}{4\omega_2} a_1 \cos(\gamma_1) - \frac{\omega_1^2}{2\omega_2} a_2 \cos(\gamma) \tag{23}$$

$$a_2' = -\frac{1}{2} v_2 a_2 + \frac{f_2}{4\omega_2} a_2 \sin(\gamma_2) - \frac{\omega_1^2}{2\omega_2} a_1 \sin(\gamma) \tag{24}$$

$$\theta_2' a_2 = \frac{\omega_1^2}{2\omega_2} a_2 - \frac{f_2}{4\omega_2} a_2 \cos(\gamma_2) - \frac{\omega_1^2}{2\omega_2} a_1 \cos(\gamma) \tag{25}$$

where $(\theta_2 - \theta_1 = \gamma)$, $(\sigma T_1 - \theta_1 = \gamma_1)$ *and* $(\sigma T_1 - \theta_2 = \gamma_2)$.

For steady solutions $a_i' = 0, \gamma_i' = 0$ and the periodic solution at the fixed points corresponding to Equations (22)-(25) is given by:

$$\frac{1}{2}(\lambda + v_1) - \frac{\omega_1^2}{2\omega_2} \frac{a_2}{a_1} \sin(\gamma) = \frac{f_1}{4\omega_2} \sin(\gamma_1) \tag{26}$$

$$-\frac{\omega_1^2}{2\omega_2} \frac{a_2}{a_1} \cos(\gamma) - \frac{\sigma}{2} + \frac{\omega_1^2}{2\omega_2} + \frac{3k_v}{8\omega_2} a_1^2 = \frac{f_1}{4\omega_2} \cos(\gamma_1) \tag{27}$$

$$\frac{1}{2} v_2 + \frac{\omega_1^2}{2\omega_2} \frac{a_1}{a_2} \sin(\gamma) = \frac{f_2}{4\omega_2} \sin(\gamma_2) \tag{28}$$

$$-\frac{\sigma}{2} + \frac{\omega_1^2}{2\omega_2} - \frac{\omega_1^2}{2\omega_2} \frac{a_1}{a_2} \cos(\gamma) = \frac{f_2}{4\omega_2} \cos(\gamma_2) \tag{29}$$

From Equations (26)-(29) we get the frequency response equation (FRE) is:

$$\left(\frac{1}{2}(\lambda+v_1)-\frac{\omega_1^2}{2\omega_2}\frac{a_2}{a_1}\sin(\gamma)\right)^2+\left(-\frac{\sigma}{2}+\frac{\omega_1^2}{2\omega_2}-\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\cos(\gamma)\right)^2-\left(\frac{f_2}{4\omega_2}\right)^2=0 \quad (30)$$

$$\left(\frac{1}{2}v_2+\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\sin(\gamma)\right)^2+\left(-\frac{\sigma}{2}+\frac{\omega_1^2}{2\omega_2}-\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\cos(\gamma)\right)^2-\left(\frac{f_2}{4\omega_2}\right)^2=0 \quad (31)$$

2.2 System with cubic control

In This subsection we study the second control using cubic negative velocity feedback the equations (1) and (2) can by written as:

$$\ddot{x}+\omega_2^2 x+\varepsilon\omega_1^2(x-y)=\varepsilon f_2 x\cos\omega t-\varepsilon v_2\dot{x}^3 \quad (32)$$

$$\ddot{y}+\omega_2^2 y+\varepsilon\omega_1^2(y-x)+\varepsilon k_v y^3+\varepsilon\lambda\dot{y}=\varepsilon f_1 y\cos\omega t-\varepsilon v_1\dot{y}^3 \quad (33)$$

where y, x are the displacements of the linear oscillators and nonlinear energy sink (NES), $\ddot{y}$ $\ddot{x}$ derivatives of y, x, $\varepsilon\lambda$ is the damping coefficient, εF_i (i=1, 2) are the amplitudes of excitation of each linear oscillator, ε is a small parameter, ω_1 , ω_2 are natural frequencies, ω is forcing frequency.

2.2.1 Mathematical analysis

Equations (32) and (33) can be solved analytically using multiple time scale perturbation technique as:
Order (ε^0):

$$(D_0^2+\omega_2^2)y_0=0 \quad (38)$$

$$(D_0^2+\omega_2^2)x_0=0 \quad (39)$$

Order (ε^1):

$$(D_0^2+\omega_2^2)y_1=-2D_0D_1y_0-\omega_1^2y_0+\omega_1^2x_0-k_vy_0^3-\lambda D_0y_0+\frac{f_1}{2}y_0\cos(\omega t)-v_1(D_0y_0)^3 \quad (40)$$

$$(D_0^2+\omega_2^2)x_1=-2D_0D_1x_0-\omega_1^2x_0+\omega_1^2y_0+\frac{f_2}{2}y_0\cos(\omega t)-v_2(D_0y_0)^3 \quad (41)$$

The general solution of Equations (6), (7) can be expressed in the form

$$x_0(T_0,T_1)=B_0e^{i\omega_2T_0}+cc \quad (42)$$

$$y_0(T_0,T_1) = A_0 e^{i\omega_2 T_0} + cc \tag{43}$$

where A_0 and B_0 are unknown function in T_1 , which can be determined by imposing the solvability condition at the next approximation order by eliminating the secular and small- divisor terms.

Substituting Equations (42), (43) into Equations (40), (41) we get:

$$\begin{aligned}(D_0^2+\omega_2^2)y_1 &= (-2i\omega_2 D_1 A_0 - \omega_1^2 A_0 + \omega_1^2 B_0 - 3k_v A_0^2 \overline{A}_0 - i\omega_2 \lambda A_0 - 3i\omega_2^3 v_1 A_0^2 \overline{A_0})e^{i\omega_2 T_0} \\ &+(2i\omega_2 D_1 \overline{A_0} - \omega_1^2 \overline{A_0} + \omega_1^2 \overline{B_0} - 3k_v \overline{A_0^2} + i\omega_2 \lambda \overline{A_0} + 3i\omega_2^3 v_1 A_0 \overline{A_0^2})e^{-i\omega_2 T_0} \\ &+(-k_v A_0^3 - i\omega_2^3 v_1 A_0^3)e^{3i\omega_2 T_0} + (-k_v \overline{A}_0^3 e + i\omega_2^3 v_1 \overline{A_0^3})^{-3i\omega_2 T_0} \\ &+\frac{f_1}{2}(A_0 e^{i(\omega+\omega_2)T_0} + \overline{A_0} e^{(\omega+\omega_2)T_0}) + \frac{f_1}{2}(A_0 e^{-i(\omega-\omega_2)T_0} + \overline{A_0} e^{i(\omega-\omega_2)T_0})\end{aligned} \tag{44}$$

$$\begin{aligned}(D_0^2+\omega_2^2)x_1 &= (-2i\omega_2 D_1 B_0 - \omega_1^2 B_0 + \omega_1^2 A_0 - i\omega_2 v_2 B_0 + 3i\omega_2^3 v_2 B_0^2 \overline{B_0})e^{i\omega_2 T_0} \\ &+(2i\omega_2 D_1 \overline{B_0} - \omega_1^2 \overline{B_0} + \omega_1^2 \overline{A_0} + i\omega_2 v_2 \overline{B_0})e^{-i\omega_2 T_0} \\ &-i\omega_2^3 v_2 B_0^3 e^{3i\omega_2 T_0} + i\omega_2^3 v_2 \overline{A_0^3} e^{-3i\omega_2 T_0} \\ &+\frac{f_2}{2}(B_0 e^{i(\omega+\omega_2)T_0} + \overline{B_0} e^{-i(\omega+\omega_2)T_0}) + \frac{f_1}{2}(B_0 e^{-i(\omega-\omega_2)T_0} + \overline{B_0} e^{i(\omega-\omega_2)T_0})\end{aligned} \tag{45}$$

Eliminating the secular terms, the general solution of Equations (44) and (45) is given by:

$$y_1(T_0,T_1) = A_1 e^{i\omega_2 T_0} + \eta_1 e^{3i\omega_2 T_0} + \xi_2 e^{i(\omega+\omega_2)T_0} + \xi_3 e^{i(\omega-\omega_2)T_0} + cc \tag{46}$$

$$x_1(T_0,T_1) = B_1 e^{i\omega_2 T_0} + \xi_4 e^{i(\omega+\omega_2)T_0} + \xi_5 e^{i(\omega-\omega_2)T_0} + \eta_4 e^{i3\omega_2 T_0} + cc \tag{47}$$

where (ζ_i , i=1…5) and A_1 , B_1 are complex function in T_1 , and cc is complex conjugate of the preceding terms.

2.2.2 Stability analysis

After numerically studying the different resonance cases and deduce the worst ones, one of the worst cases has been chosen to study the system stability. The selected resonance case is the Sub-harmonic case ($\omega = 2\omega_2$). In this case we introduce the detuning parameter σ according to:

$$\omega = 2\omega_2 + \sigma T_1 \tag{48}$$

where σ is the detuning parameter .Also for stability investigation, the analysis is limited to the first approximation .

Substituting equations (48) into Equations (44), (45) and eliminating the secular terms leads to the solvability conditions

$$(-2i\omega_2 D_1 A_0 - \omega_1^2 A_0 + \omega_1^2 B_0 - 3k_v A_0^2 \overline{A}_0 - i\omega_2 \lambda A - 3i\omega_2^3 v_1 A_0^2 \overline{A}_0 + \frac{f_1}{2}\overline{A}_0 e^{i\sigma T_1}) = 0 \quad (49)$$

$$(-2i\omega_2 D_1 B_0 - \omega_1^2 B_0 + \omega_1^2 A_0 + 3i\omega_2^3 v_2 B_0^2 \overline{B_0} + \frac{f_2}{2}\overline{B}_0 e^{i\sigma T_1}) = 0 \quad (50)$$

To analyze the solution of Equations (18) and (19), it is convenient to express A_0 in the polar form as:

$$A_0(T_1) = \frac{1}{2} a_1(T_1) e^{i\theta_1(T_1)} \quad (51)$$

$$B_0(T_1) = \frac{1}{2} a_2(T_1) e^{i\theta_2(T_1)} \quad (52)$$

where $a_i(i=1,2), \theta_i(i=1,2)$ is unknown real-valued function. Inserting Equations (51), (52) into Equations (50), (49) and separating the real and imaginary parts we have the following:

$$a_1' = \frac{3\omega_2^2}{8} v_1 a_1^3 - \frac{\lambda}{2} a_1 + \frac{f_1}{4\omega_2} a_1 \sin(\gamma_1) + \frac{\omega_1^2}{2\omega_2} a_2 \sin(\gamma) \quad (53)$$

$$\theta_1' a_1 = \frac{\omega_1^2}{2\omega_2} a_1 + \frac{3k_v}{8\omega_2} a_1^3 - \frac{f_1}{4\omega_2} a_1 \cos(\gamma_1) - \frac{\omega_1^2}{2\omega_2} a_2 \cos(\gamma) \quad (54)$$

$$a_2' = \frac{3\omega_2^2}{8} v_2 a_2^3 - \frac{f_2}{4\omega_2} a_2 \sin(\gamma_2) - \frac{\omega_1^2}{2\omega_2} a_1 \sin(\gamma) \quad (55)$$

$$\theta_2' a_2 = \frac{\omega_1^2}{2\omega_2} a_2 - \frac{f_2}{4\omega_2} a_2 \cos(\gamma_2) - \frac{\omega_1^2}{2\omega_2} a_1 \cos(\gamma) \quad (56)$$

where $(\theta_2 - \theta_1 = \gamma), (\sigma T_1 - \theta_1 = \gamma_1)$ *and* $(\sigma T_1 - \theta_2 = \gamma_2)$.

For steady solutions $a_i' = 0, \gamma_i' = 0$ and the periodic solution at the fixed points corresponding to Equations (53)-(56) is given by:

$$\frac{1}{2}\lambda - \frac{3\omega_2^3}{8} v_1 a_1^2 - \frac{\omega_1^2}{2\omega_2}\frac{a_2}{a_1}\sin(\gamma) = \frac{f_1}{4\omega_2}\sin(\gamma_1) \quad (57)$$

$$-\frac{\omega_1^2}{2\omega_2}\frac{a_2}{a_1}\cos(\gamma) - \frac{\sigma}{2} + \frac{\omega_1^2}{2\omega_2} + \frac{3k_v}{8\omega_2} a_1^2 = \frac{f_1}{4\omega_2}\cos(\gamma_1) \quad (58)$$

$$-\frac{3\omega_2^2}{8}v_2a_2^2+\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\sin(\gamma)=\frac{f_2}{4\omega_2}\sin(\gamma_2) \tag{59}$$

$$-\frac{\sigma}{2}+\frac{\omega_1^2}{2\omega_2}-\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\cos(\gamma)=\frac{f_2}{4\omega_2}\cos(\gamma_2) \tag{60}$$

Form Equations (57)-(60) we get the corresponding frequency response equation (FRE) is:

$$\left(\frac{1}{2}\lambda-\frac{3\omega_2^3}{8}v_1a_1^2-\frac{\omega_1^2}{2\omega_2}\frac{a_2}{a_1}\sin(\gamma)\right)^2+\left(-\frac{\omega_1^2}{2\omega_2}\frac{a_2}{a_1}\cos(\gamma)-\frac{\sigma}{2}+\frac{\omega_1^2}{2\omega_2}+\frac{3k_v}{8\omega_2}a_1^2\right)^2-\left(\frac{f_1}{4\omega_2}\right)^2=0 \tag{61}$$

$$\left(-\frac{3\omega_2^2}{8}v_2a_2^2+\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\sin(\gamma)\right)^2+\left(-\frac{\sigma}{2}+\frac{\omega_1^2}{2\omega_2}-\frac{\omega_1^2}{2\omega_2}\frac{a_1}{a_2}\cos(\gamma)\right)^2-\left(\frac{f_2}{4\omega_2}\right)^2=0 \tag{62}$$

3 Results and Discussion

To study behavior of the main system numerically the (Rung-Kutta method) of the nonlinear system, given by Equations (1) and (2) at basic without absorber, the Sub-harmonic resonance case ($\omega \simeq 2\omega_2$) is obtained as shown in Figs. (2)- (6). These solutions are obtained at selected values ($\omega_2 = 0.1$, $\omega \simeq 2\omega_2$).

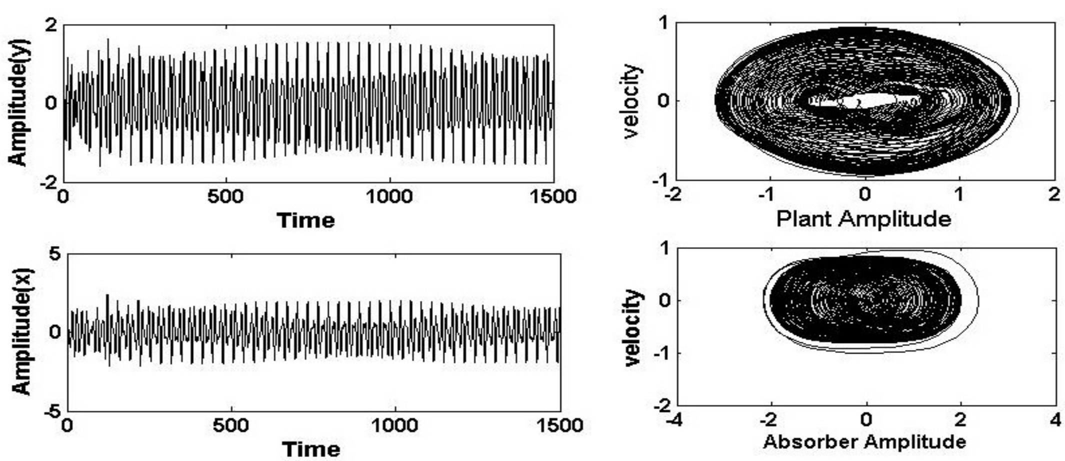


Fig. 2. Response of the system without absorber at basic case

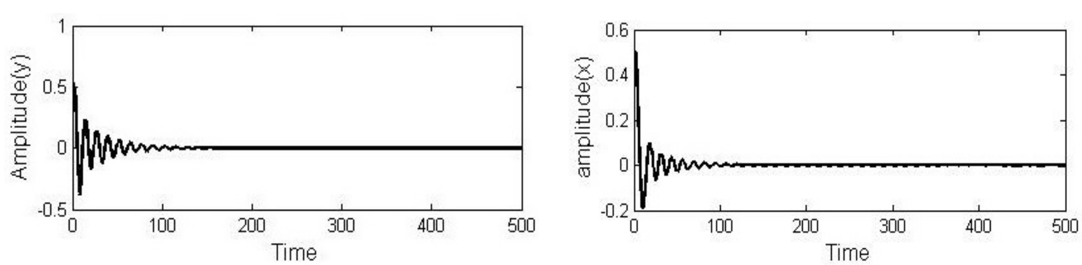


Fig. 3. Response of the system at basic with linear control

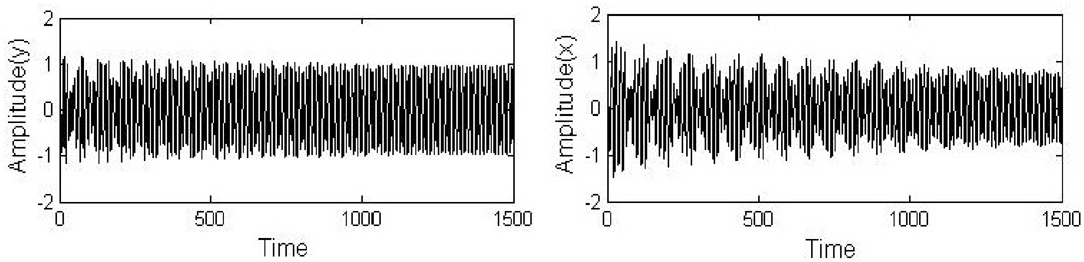


Fig. 4. Response of the system at basic with cubic control

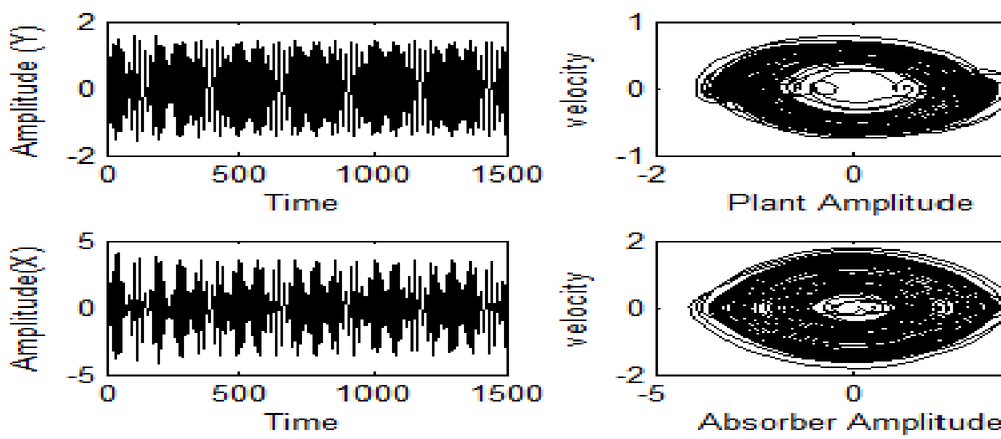


Fig. 5. Response of the system at worst case at basic without absorber ($\omega \simeq 2\omega_2$)

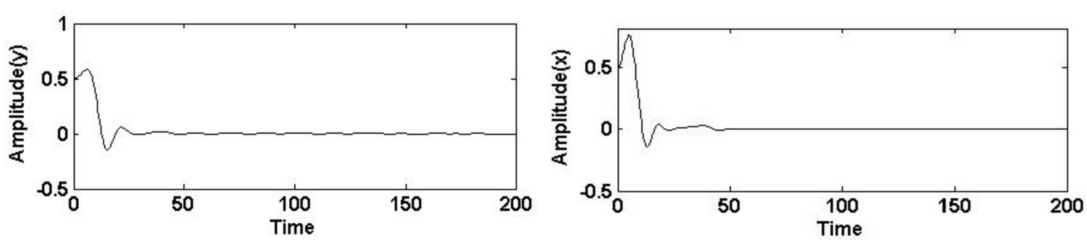


Fig. 6. Response of the system at Worst case at basic with linear control ($\omega \simeq 2\omega_2$)

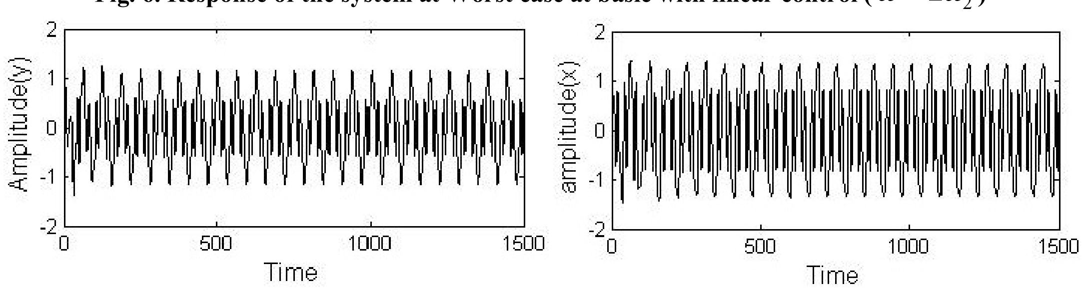


Fig. 7. Response of the system at Worst case at basic with cubic control ($\omega \simeq 2\omega_2$)

Fig. (2) Show that study of amplitude on the main system in the basic case without absorber. Fig. (3) And (4) Show that in each of them we study the amplitude in the system in the basic case. First, we study the effect of linear control where the maximum stability is reached (zero), the second is the effect of cubic control where the amplitude remains unstable (up to approximately 1.4). In comparison between the effect of linear control and cubic control on the system it is clear that the use of linear control has better results than the use of cubic control in controlling the intensity of vibration and dynamic chaos on the system. Fig. (5) Show that study of amplitude on the main system in the worst case sub harmonic ($\omega \simeq 2\omega_2$) without absorber it is also shown that these are the worst resonance cases such that the oscillations of the system and absorber have multi-limit cycle and increasing dynamic chaos and this is the condition that must be controlled using the control to fade the vibration and pressure damage. Fig. (6) And (7) Show vibration control in the system is controlled to compare the effect of each of their ability to control the worst. First, we study the effect of linear control where the maximum stability is reached (zero), the second is the effect of cubic control where the amplitude remains unstable (up to approximately 1.7). In comparison between the effect of linear control and cubic control on the system it is clear that the use of linear control has better results than the use of cubic control in controlling the intensity of vibration and dynamic chaos on the system.

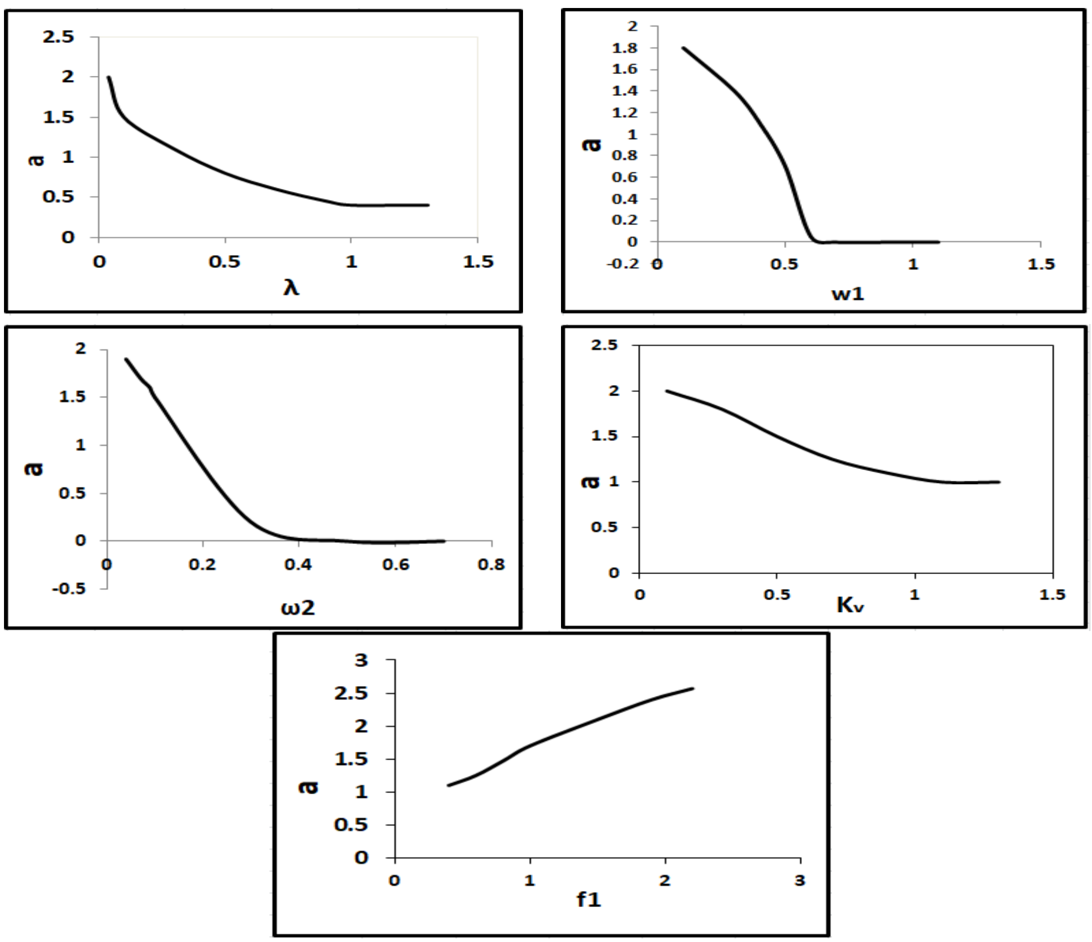


Fig. 8. Effect of parameter on amplitude of the system on the basic case

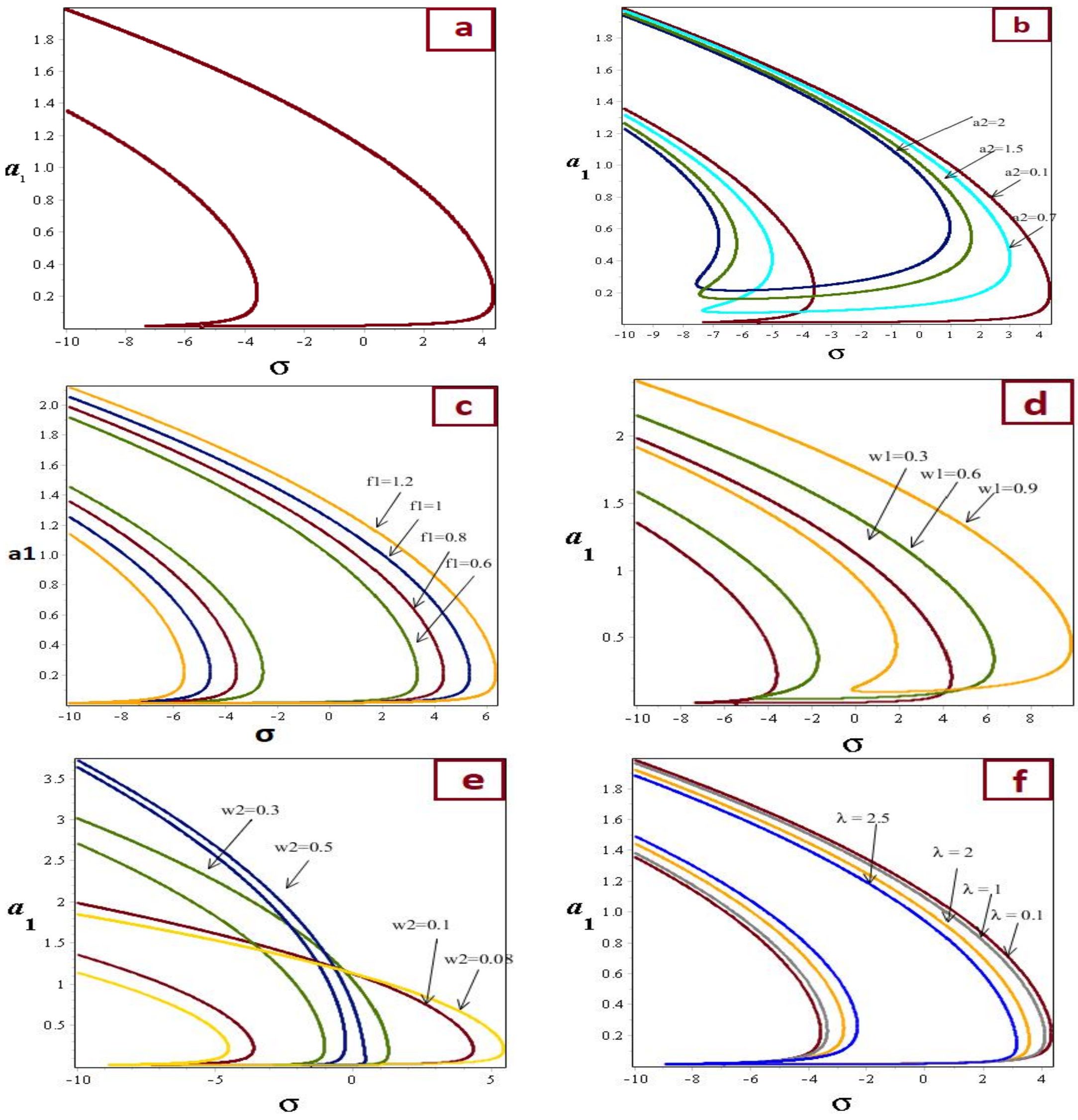
Fig.8 show effect of different parameter on the main system without absorber .can see amplitude increasing as f_1 is increased .Also when decreasing the values ω_2 , ω_1 , k_v and λ are increasing as shown.

4 Frequency and Force Response Curve

The frequency equation is represented graphically by using the numerical methods. The frequency response equation is nonlinear algebraic equation, which are solved numerically by using Newton Raphson method

4.1 Frequency response curve from system with linear absorber

The frequency response equation (18) (19) is nonlinear algebraic equation, the results are shown in figures (9, 10), for the steady state amplitudes a_1 and a_2 against parameter σ



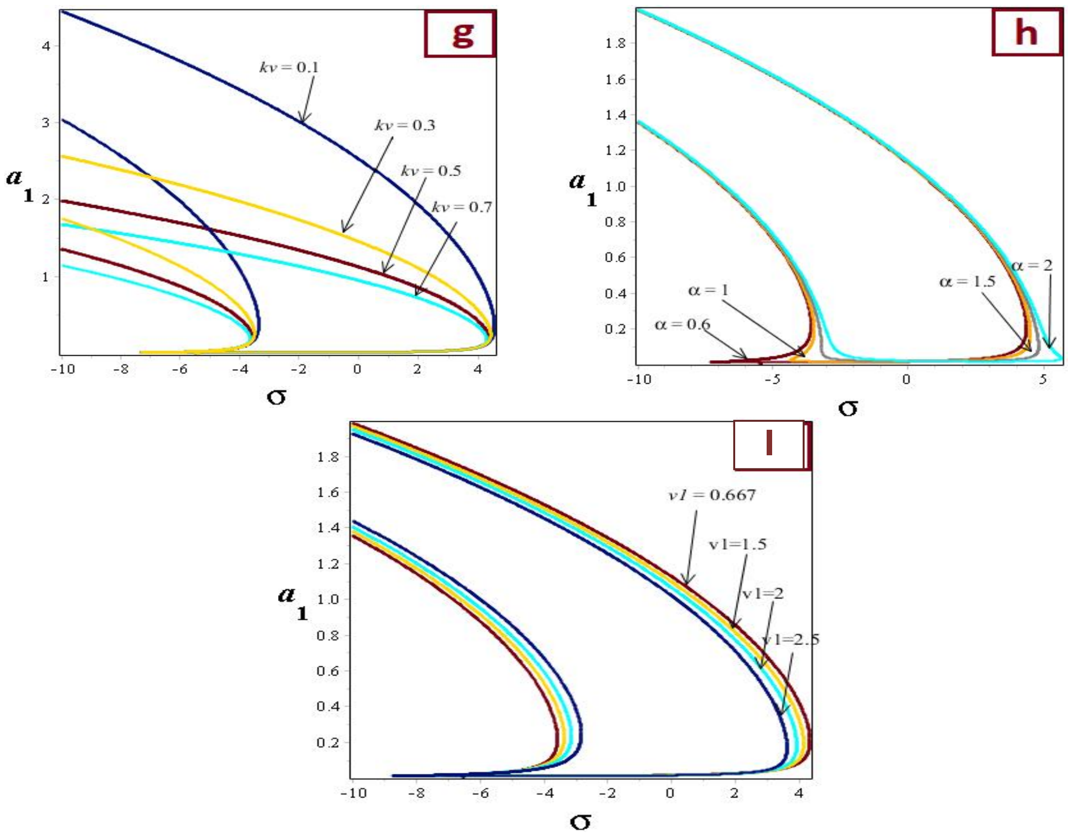
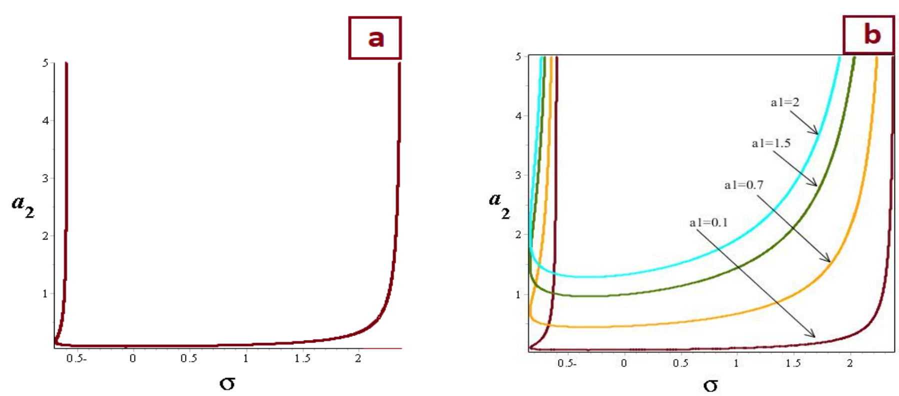


Fig. 9. Frequency response curves at the case, $a_1 \neq 0$, $a_2 \neq 0$ amplitude a_1 against σ

Fig. 9 show that the steady-state amplitude a1 increased as f_1, ω_1 and ω_2 increased. Also when decreasing the values λ , k_v , v_1 and α are increasing, the frequency response curve is bend to up with increasing values ω_1 and ω_2 also curve bend to down with decreasing values λ and kv as shown in Fig. 10. The frequency response curve values will stable when ω_1 <0.3 and λ<0.1.



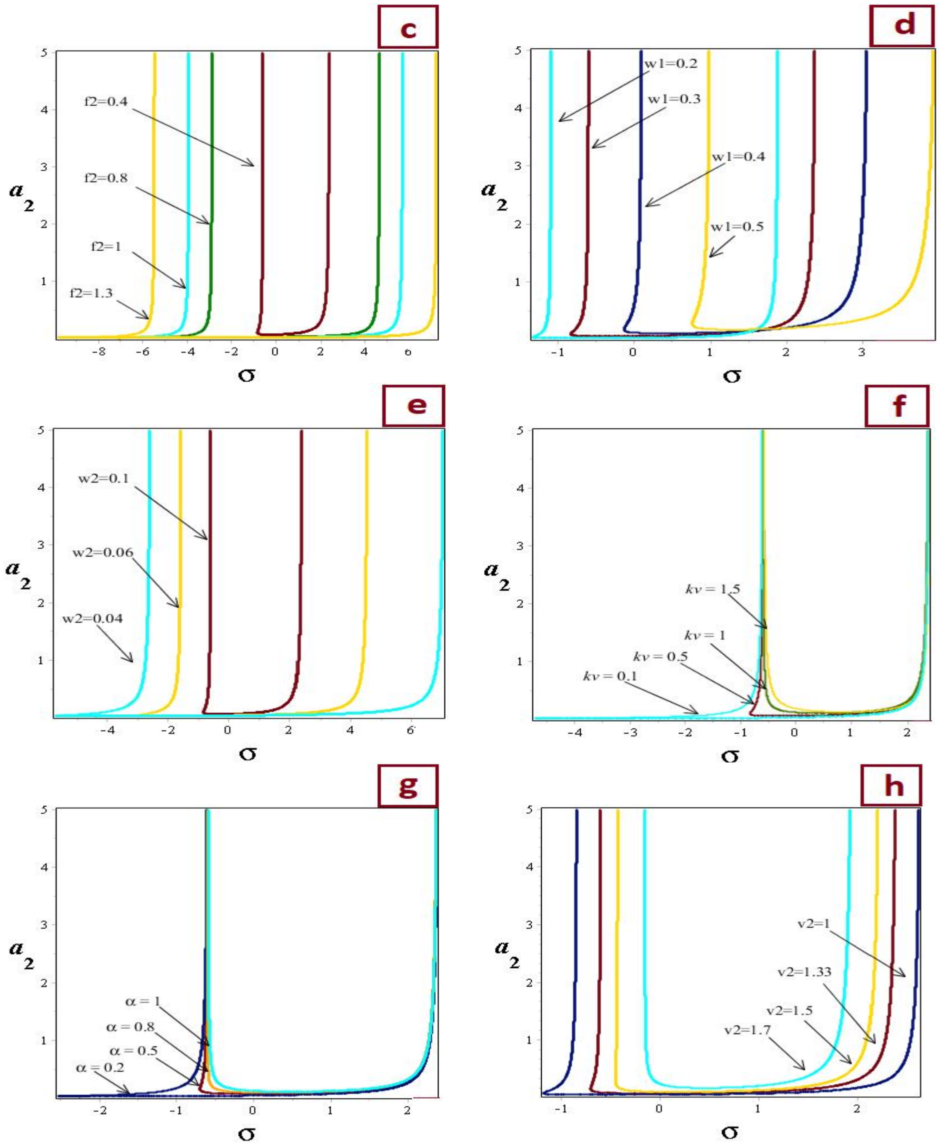


Fig. 10. Frequency response curves at the case, $a_1 \neq 0$, $a_2 \neq 0$ the amplitude a_2 against σ

Fig. 10 shown that the steady-state amplitude a1 increased as k_v, a_1, α, ω_1 and v_2 increased. Also when decreasing the values f_2 and ω_2 are increasing, the frequency response curve is bending to up with increasing values v_2, a_1 and ω_1 as shown in fig. The frequency response curve values will stable when ω_2 >0.1.

4.2 Frequency response curve from system with cubic absorber

The frequency response equations (49), (50) is nonlinear algebraic equation, the results are shown in figures (11, 12), for the steady state amplitudes a_1 and a_2 against parameters σ

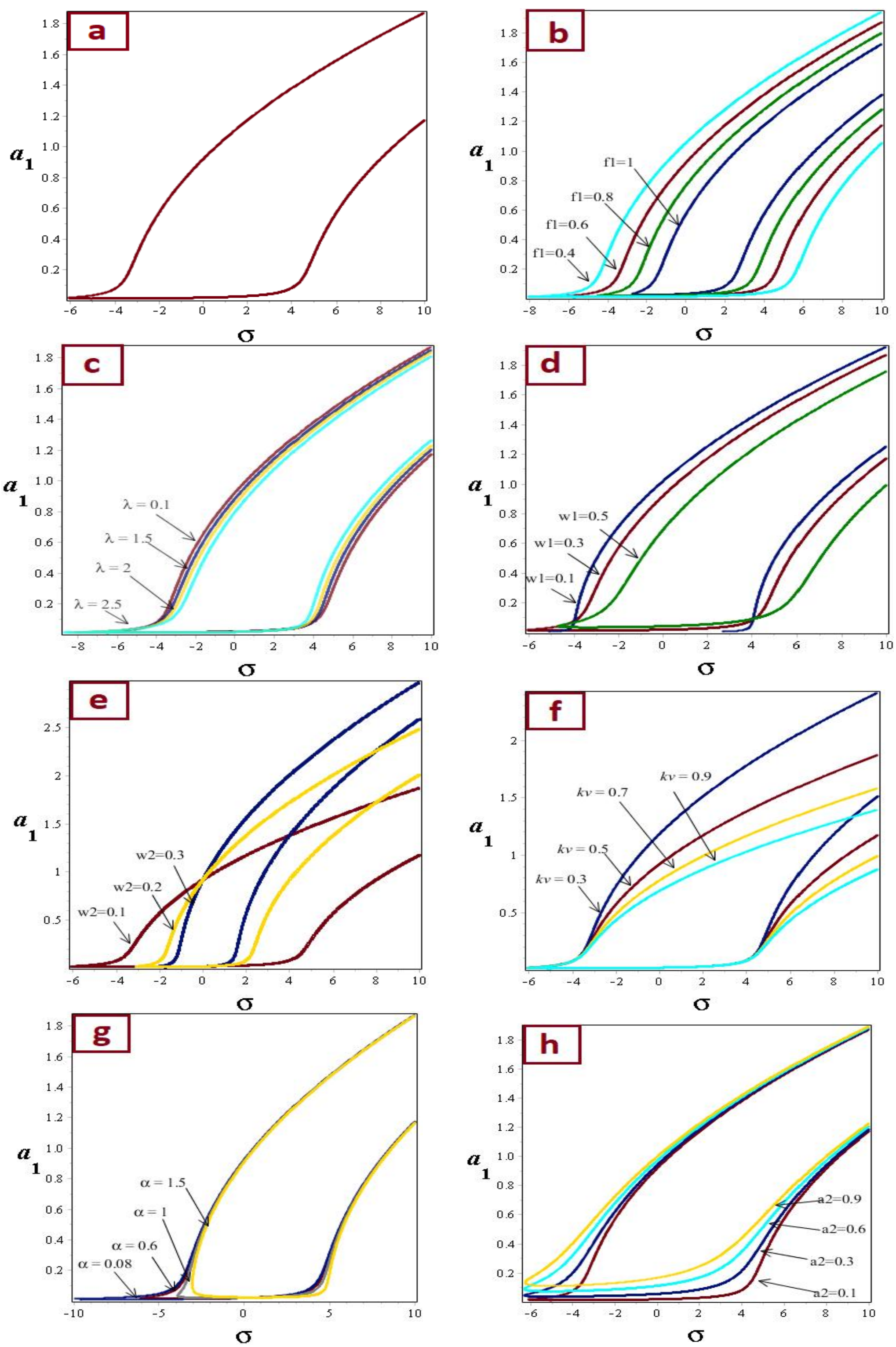


Fig. 11. Frequency response curves at case, $a_1 \neq 0$, $a_2 \neq 0$ the amplitude a_1 against σ

Fig. 11 show that the steady-state amplitude a_1 increased as a_2 and ω_2 increased. Also when decreasing the values λ, k_v , f_1 and ω1 are increasing, the frequency response curve is bending to up left with increasing values a_2 and ω_2 also curve bend to down with decreasing values λ, ω_2 and k_v as shown in Fig. 12. The frequency response curve not affected by increasing or decreasing values f_2 , v_2 and be stable when λ<0.1.

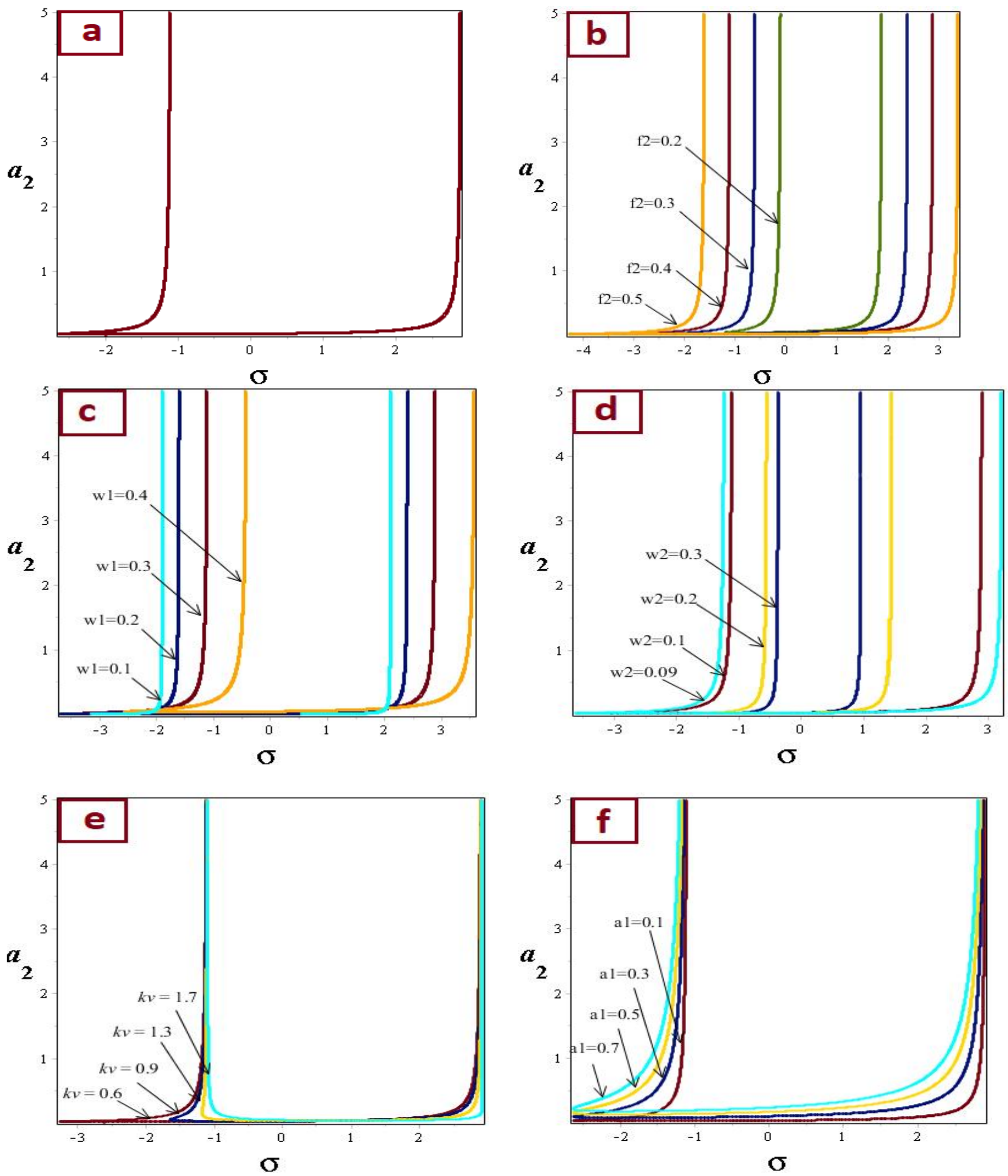


Fig. 12. Frequency response curves at the case, $a_1 \neq 0, a_2 \neq 0$ the amplitude a_2 against σ

Fig. 12 shown that the steady-state amplitude a_2 increased as a_1, k_v, ω_2 and ω_1 increased. Also when decreasing the values f_2 is increasing, the frequency response curve is bending to up left with increasing values a_1 and ω_2. The frequency response curve not affected by increasing or decreasing values f_1, λ, α and be stable when k_v <0.6.

5 Conclusions

Different controls (linear and cubic) feedback are applied on the 2dof system of linear coupled oscillators and nonlinear energy sinks. The method of multiple time scales is perturbation and the technique is applied to obtain the periodic response equation near the selected resonance case. Also both frequency response equation and results based on the present investigation the above study the following conclusions:

1. The worst behavior of the main system occurs at sub-harmonic resonance ($\omega \simeq 2\omega_2$) when the absorbers aren't effective.
2. The behavior of amplitude on the main system is more stable and control vibration is almost nonexistent in linear control than cubic control.
3. The steady state amplitude of the main system is monotonic increasing function f_1, ω_1 and ω_2 on steady state amplitude a_1 against **σ**, monotonic increasing function k_v, a_1, α, ω_1 and v2 on steady state amplitude a_2 against **σ** on effect linear control.
4. The steady state amplitude of the main system is monotonic decreasing function ω_1 and ω_2 on steady state amplitude a_1 against **σ**, monotonic decreasing function f_2 and ω_2 on steady state amplitude a_2 against **σ** on effect linear control.
5. The steady state amplitude of the main system is monotonic increasing function a_2 and ω_2 on steady state amplitude a_1 against **σ**, monotonic increasing function k_v, ω_2 and ω_1 on steady state amplitude a_2 against **σ** on effect cubic control.
6. The steady state amplitude of the main system is monotonic decreasing function λ, k_v, f_1 and ω_1 on steady state amplitude a_1 against **σ**, monotonic decreasing function f_2 on steady state amplitude a_2 against **σ** on effect cubic control.

Competing Interests

Authors have declared that no competing interests exist.

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APPENDIX

$$\xi_1 = \frac{k_v}{8\omega_2^2} A_0^3, \quad \xi_2 = \frac{f_1 A_0}{2(\omega_2^2 - (\omega + \omega_2)^2)}, \quad \xi_3 = \frac{f_1 \overline{A_0}}{2(\omega_2^2 - (\omega - \omega_2)^2)}$$

$$\xi_4 = \frac{f_2 B_0}{2(\omega_2^2 - (\omega + \omega_2)^2)}, \quad \xi_5 = \frac{f_2 \overline{B_0}}{2(\omega_2^2 - (\omega - \omega_2)^2)}, \quad \eta_4 = \frac{v_2 \omega_2^3}{8\omega_2^2} B_0^3, \quad \eta_1 = (\frac{k_v}{8\omega_2^2} A_0^3 + \frac{v_1 \omega_2^3}{8\omega_2^2} A_0^3)$$

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