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Fixed Points of E-Contraction in Controlled Metric Spaces

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Authors' contributions

This work was carried out in collaboration among all authors. All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript. All authors read and approved the final manuscript.

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Abstract

The goal of this study is to prove a fixed-point theorem for E-contraction in a completely controlled metric space. Many previous findings in the literature are extended/ generalized by our findings. We also present examples that demonstrate the utility of these findings.

Keywords: Fixed point theory; E-contraction; controlled metric space.

MSC: 47H10; 54H25

1 Introduction and Preliminaries

"The notion of E-contraction was introduced by Fulga and Proca [1]. Later, this concept has been improved by several authors, e.g.", [2-4].

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Dass and Gupta [5] established “first fixed point theorem for rational contractive type conditions in metric space”.

Theorem 1.1 (see [5]) Let (Y, d) be a complete metric space, and let $\mathcal{T}: Y \to Y$ be a self-mapping. If there exist $\alpha, \beta \in [0, 1)$ with $\alpha + \beta < 1$ such that

$$d(\mathcal{T}\xi, \mathcal{T}\nu) \leq \alpha d(\xi, \nu) + \beta \frac{[1 + d(\xi, \mathcal{T}\xi)]d(\nu, \mathcal{T}\nu)}{1 + d(\xi, \nu)} \tag{1.1}$$

for all $\xi, \nu \in Y$, then $\mathcal{T}$ has a unique fixed point $\xi^* \in Y$.

Nazam et al. [6] proved “a real generalization of Dass-Gupta fixed point theorem in the frame work of dualistic partial metric spaces”.

Czerwik [7] reintroduced “a new class of generalized metric spaces, called as b-metric spaces, as generalizations of metric spaces”.

Definition 1 (see [7]) Let Y be a nonempty set and $s \geq 1$. A function $d_b: Y \times Y \longrightarrow [0, \infty)$ is said to be a b - metric if for all $\xi, \nu, \omega \in Y$,

(b1). $d_b(\xi, \nu) = 0$ iff $\xi = \nu$
(b2). $d_b(\xi, \nu) = d_b(\nu, \xi)$for all $\xi, \nu \in Y$
(b3). $d_b(\xi, \omega) \leq s[d_b(\xi, \nu) + d_b(\nu, \omega)]$

“The pair (Y, d_b) is then called a b-metric space. Subsequently, many fixed-point results on such spaces were given” (see [8–13]).

Kamran et al. [14] initiated “the concept of extended b-metric spaces”.

Definition 2 (see [8]) Let Y be a nonempty set and $p: Y \times Y \longrightarrow [1, \infty)$be a function. A function $d_e: Y \times Y \longrightarrow [0, \infty)$is called an extended b -metric if for all $\xi, \nu, \omega \in Y$,

(e1). $d_e(\xi, \nu) = 0$ iff $\xi = \nu$
(e2). $d_e(\xi, \nu) = d_e(\nu, \xi)$for all $\xi, \nu \in Y$
(e3). $d_e(\xi, \omega) \leq p(\xi, \omega)[d_e(\xi, \nu) + d_e(\nu, \omega)]$

The pair$(Y, d_e)$is called an extended b-metric space.

“Recently, a new kind of a generalized b-metric space was introduced” by Mlaiki et al. [15].

Definition 3 (see [15]) Let Y be a nonempty set and $p: Y \times Y \longrightarrow [1, \infty)$be a function. A function $d_c: Y \times Y \longrightarrow [0, \infty)$is called a controlled metric if for all $\xi, \nu, \omega \in Y$,

(c1). $d_c(\xi, \nu) = 0$ iff $\xi = \nu$
(c2). $d_c(\xi, \nu) = d_c(\nu, \xi)$for all $\xi, \nu \in Y$
(c3). $d_c(\xi, \omega) \leq p(\xi, \nu)d_c(\xi, \nu) + p(\nu, \omega)d_c(\nu, \omega)$

The pair$(Y, d_c)$is called a controlled metric space (see also [16]).

The Cauchy and convergent sequences in controlled metric type spaces are defined in this way [17-23]

Definition 4 (see [15]) Let(Y, d_c)be a controlled metric space and $\{\xi_n\}_{n\geq 0}$be a sequence in D. Then,

1. The sequence $\{\xi_n\}$ converges to some ξ in Y; if for every $\varepsilon > 0$, there exists $N = N(\varepsilon) \in \mathbb{N}$ such that $d_c(\xi_n, \xi) < \varepsilon$ for all $n \geq N$. In this case, we write$\lim_{n\to\infty} \xi_n = \xi$.
2. The sequence $\{\xi_n\}$ is Cauchy; if for every $\varepsilon > 0$,there exists $N = N(\varepsilon) \in \mathbb{N}$such that $d_c(\xi_n, \xi_m) < \varepsilon$ for all $n, m \geq N$.
3. The controlled metric space $(Y, d_c)$is called complete if every Cauchy sequence is convergent.

Definition 5 (see [15]) Let (Y, d_c)be a controlled metric space. Let $\xi \in Y$and $\varepsilon > 0$.

1. The open ball $B(\xi, \varepsilon)$ is

   $B(\xi, \varepsilon) = \{\nu \in Y: d_c(\nu, \xi) < \varepsilon\}$.

2. The mapping $\Gamma: Y \longrightarrow Y$ is said to be continuous at $\xi \in Y$; if for all $\varepsilon > 0$, there exists $\delta > 0$ such that$\Gamma\big(B(\xi, \varepsilon)\big) \subseteq B(\Gamma\xi, \varepsilon)$.

This paper's main objective is to propose a fixed-point theorem for E-contractions in the context of complete controlled metric spaces. Our finding broadens and generalises a few established findings in the literature [24-32]. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

2 Main Results

The following theorem is our main result.

Theorem 2.1 Let (Y, d_c)be a complete controlled metric space and $\Gamma: Y \longrightarrow Y$ be a mapping such that

$$d_c(\Gamma\xi, \Gamma\nu) \leq \delta[d_c(\xi, \nu) + |d_c(\xi, \Gamma\xi) - d_c(\nu, \Gamma\nu)|] \tag{2.1}$$

for all $\xi, \nu \in Y$, where $0 \leq \delta < 1$. For $\xi_0 \in Y$ and each n, we let $\xi_n = \Gamma^n \xi_0$. If

$$\sup_{m\geq 1} \lim_{i\to\infty} \frac{p(\xi_{i+1},\xi_{i+2})p(\xi_{i+1},\xi_m)}{p(\xi_i,\xi_{i+1})} < \frac{1+\delta}{2\delta} \tag{2.2}$$

and $\lim_{n\to\infty} p(\xi_n, \xi)$ and $\lim_{n\to\infty} p(\xi, \xi_n)$ exist, are finite, and $\delta \lim_{n\to\infty} p(\xi, \xi_n) < 1$ for every $\xi \in Y$, then Γ possesses a unique fixed point.

Proof Let $\xi_0 \in Y$ be an initial point. Define the sequence $\{\xi_n\}$ by $\xi_{n+1} = \Gamma\xi_n, \forall\, n \in \mathbb{N}$. Obviously, if there exists $n_0 \in \mathbb{N}$ such that $\xi_{n_0+1} = \xi_{n_0}$, then $\Gamma\xi_{n_0} = \xi_{n_0}$, and the proof is finished. Thus, we assume that $\xi_{n+1} \neq \xi_n$ for each$n \in \mathbb{N}$. Thus, by (2.1), we have

$$\begin{aligned} d_c(\xi_n, \xi_{n+1}) &= d_c(\Gamma\xi_{n-1}, \Gamma\xi_n) \\ &\leq \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \Gamma\xi_{n-1}) - d_c(\xi_n, \Gamma\xi_n)| \\ &= \delta d_c(\xi_{n-1}, \xi_n) + \delta|d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})| \end{aligned} \tag{2.3}$$

If $d_c(\xi_{n-1}, \xi_n) \leq d_c(\xi_n, \xi_{n+1})$ for some n, then from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) - d_c(\xi_{n-1}, \xi_n) + d_c(\xi_n, \xi_{n+1})] = d_c(\xi_n, \xi_{n+1})$$

which is a contradiction. Hence $d_c(\xi_{n-1}, \xi_n) > d_c(\xi_n, \xi_{n+1})$, and so from (2.3), we have

$$d_c(\xi_n, \xi_{n+1}) \leq \delta[d_c(\xi_{n-1}, \xi_n) + d_c(\xi_{n-1}, \xi_n) - d_c(\xi_n, \xi_{n+1})]$$

The last inequality gives

$$d_c(\xi_n, \xi_{n+1}) \leq \frac{2\delta}{1+\delta} d_c(\xi_{n-1}, \xi_n) \tag{2.3}$$

Let $\lambda = \frac{2\delta}{1+\delta} < 1$. Thus, we have

$$d_c(\xi_n, \xi_{n+1}) \leq \lambda d_c(\xi_{n-1}, \xi_n) \leq \lambda^2 d_c(\xi_{n-2}, \xi_{n-1}) \leq \cdots \leq \lambda^n d_c(\xi_0, \xi_1) \tag{2.4}$$

For all $n, m \in \mathbb{N}$ and $n < m$, we have

$$\begin{aligned}
& d_c(\xi_n, \xi_m) \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m) d_c(\xi_{n+1}, \xi_m) \\
& \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m) p(\xi_{n+1}, \xi_{n+2}) d_c(\xi_{n+1}, \xi_{n+2}) \\
& + p(\xi_{n+1}, \xi_m) p(\xi_{n+2}, \xi_m) d_c(\xi_{n+2}, \xi_m) \\
& \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) + p(\xi_{n+1}, \xi_m) p(\xi_{n+1}, \xi_{n+2}) d_c(\xi_{n+1}, \xi_{n+2}) \\
& + p(\xi_{n+1}, \xi_m) p(\xi_{n+2}, \xi_m) p(\xi_{n+2}, \xi_{n+3}) d_c(\xi_{n+2}, \xi_{n+3}) \\
& + p(\xi_{n+1}, \xi_m) p(\xi_{n+2}, \xi_m) p(\xi_{n+3}, \xi_m) d_c(\xi_{n+3}, \xi_m) \\
& \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) \\
& + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) d_c(\xi_i, \xi_{i+1}) \\
& + \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m)\, d_c(\xi_{m-1}, \xi_m)
\end{aligned} \tag{2.5}$$

This implies that

$$\begin{aligned}
& d_c(\xi_n, \xi_m) \leq p(\xi_n, \xi_{n+1}) d_c(\xi_n, \xi_{n+1}) \\
& + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) d_c(\xi_i, \xi_{i+1}) \\
& + \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m)\, d_c(\xi_{m-1}, \xi_m) \\
& \leq p(\xi_n, \xi_{n+1}) \lambda^n d_c(\xi_0, \xi_1) \\
& + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1) \\
& + \prod_{i=n+1}^{m-1} p(\xi_j, \xi_m)\, \lambda^{m-1} d_c(\xi_0, \xi_1) \\
& \leq p(\xi_n, \xi_{n+1}) \lambda^n d_c(\xi_0, \xi_1) \\
& + \sum_{i=n+1}^{m-1} \left(\prod_{j=n+1}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1)
\end{aligned} \tag{2.6}$$

Let

$$\eta_r = \sum_{i=0}^{r} \left(\prod_{j=0}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1) \tag{2.7}$$

Consider

$$\mu_i = \sum_{i=0}^{r} \left(\prod_{j=0}^{i} p(\xi_j, \xi_m) \right) p(\xi_i, \xi_{i+1}) \lambda^i d_c(\xi_0, \xi_1) \tag{2.8}$$

Then by condition (2.2) and by the ratio test, the series $\sum_i \mu_i$ is convergent. That is, $\lim_{n \to \infty} \eta_n$exists. Hence, the sequence $\{\eta_n\}$ is Cauchy. Now, by (2.6), we have

$$d_c(\xi_n, \xi_m) \leq d_c(\xi_0, \xi_1) [\lambda^n p(\xi_n, \xi_{n+1}) + (\eta_{m-1} - \eta_n)] \tag{2.9}$$

Note that $p(\xi, \nu) \geq 1$. Letting $n, m \longrightarrow \infty$ in (2.9), we obtain

$$\lim_{n,m \to \infty} d_c(\xi_n, \xi_m) = 0 \tag{2.10}$$

This shows that the sequence $\{\xi_n\}$is Cauchy in the complete controlled metric space (Y, d_c). Thus, there is some $\xi^* \in Y$.So that

$$\lim_{n \to \infty} d_c(\xi_n, \xi^*) = 0; \tag{2.11}$$

that is, $\xi_n \longrightarrow \xi^*$as $n \longrightarrow \infty$. Now, we will prove that $\xi^*$is a fixed point of Γ. By (2.1) and condition (iii), we have

$$\begin{aligned} & d_c(\xi^*,\Gamma\xi^*) \leq p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) + p(\xi_{n+1},\Gamma\xi^*)d_c(\xi_{n+1},\Gamma\xi^*) \\ & = p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) + p(\xi_{n+1},\Gamma\xi^*)d_c(\Gamma\xi_n,\Gamma\xi^*) \\ & \leq p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) \\ & + p(\xi_{n+1},\Gamma\xi^*)\delta[d_c(\xi_n,\xi^*) + |d_c(\xi_n,\Gamma\xi_n) - d_c(\xi^*,\Gamma\xi^*)|] \\ & \leq p(\xi^*,\xi_{n+1})d_c(\xi^*,\xi_{n+1}) \\ & + p(\xi_{n+1},\Gamma\xi^*)\delta[d_c(\xi_n,\xi^*) + |d_c(\xi_n,\xi_{n+1}) - d_c(\xi^*,\Gamma\xi^*)|] \end{aligned} \tag{2.12}$$

Since $\lim_{n\to\infty} p(\xi_n,\xi)$and $\lim_{n\to\infty} p(\xi,\xi_n)$ exist, are finite, by (2.10), (2.11), we have

$$d_c(\xi^*,\Gamma\xi^*) \leq [\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1},\Gamma\xi^*)]d_c(\xi^*,\Gamma\xi^*) \tag{2.13}$$

Suppose that $\xi^* \neq \Gamma\xi^*$, having in mind that $[\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1},\Gamma\xi^*)] < 1$, so

$$0 < d_c(\xi^*,\Gamma\xi^*) \leq [\delta \lim_{n\longrightarrow\infty} p(\xi_{n+1},\Gamma\xi^*)]d_c(\xi^*,\Gamma\xi^*) < d_c(\xi^*,\Gamma\xi^*) \tag{2.14}$$

This is a contradiction. Thus, we must have $\xi^* = \Gamma\xi^*$. Next, we show that ξ^* is unique. Let ν^* be another fixed point of Γ in Y, then $\Gamma\nu^* = \nu^*$. And so, by (2.1), we have

$$\begin{aligned} & d_c(\xi^*,\nu^*) = d_c(\Gamma\xi^*,\Gamma\nu^*) \\ & \leq \delta[d_c(\xi^*,\nu^*) + |d_c(\xi^*,\Gamma\xi^*) - d_c(\nu^*,\Gamma\nu^*)|] \\ & = \delta[d_c(\xi^*,\nu^*) + |d_c(\xi^*,\xi^*) - d_c(\nu^*,\nu^*)|] \\ & = \delta d_c(\xi^*,\nu^*) (2.15) \end{aligned}$$

This is a contradiction. Thus, $\xi^* = \nu^*$. It completes the proof.

3 Examples

Now we furnish some examples to demonstrate the validity of the hypotheses of generality of our result.

Example 3.1 Let$Y = \{0,1,2\}$. Take the controlled metric d_c defined as

$$\begin{aligned} & d_c(0,0) = d_c(1,1) = d_c(2,2) = 0, \\ & d_c(0,1) = d_c(1,0) = \frac{1}{2}, d_c(0,2) = d_c(2,0) = \frac{11}{20}, d_c(1,2) = d_c(2,1) = \frac{3}{20}, \end{aligned}$$

where $p: Y \times Y \longrightarrow [1,\infty)$ is symmetric such that

$$p(0,0) = p(1,1) = p(2,2) = p(1,2) = 1, p(0,2) = 2, p(0,1) = \frac{3}{2}$$

Given $\Gamma : Y \longrightarrow Y$ as

$\Gamma 0 = 2$ and $\Gamma 1 = \Gamma 2 = 1$.
If $\gamma = \frac{2}{3}$. Then

$$\lambda = \frac{2\gamma}{1+\gamma} == \frac{\frac{4}{3}}{1+\frac{2}{3}} = \frac{4}{5} < 1,$$

Take $\xi_0 = 0$, then $\xi_1 = 2$, and $\xi_n = 1$, for all $n \geq 2$, we have$\lim_{n\to\infty} p(\xi_n,\xi)$ and $\lim_{n\to\infty} p(\xi,\xi_n)$ exist, are finite, and $\gamma \lim_{n\to\infty} p(\xi,\xi_n) < 1$ for every $\xi \in Y$. Also

$$\sup_{m\geq 1}\lim_{i\to\infty}\frac{p(\xi_{i+1},\xi_{i+2})p(\xi_{i+1},\xi_m)}{p(\xi_i,\xi_{i+1})} = 1 < \frac{5}{4} = \lambda^{-1}$$

We consider the following cases.

(1) Let $\xi = \nu = 0$,then

$$d_c(\Gamma\xi,\Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(2) Let $\xi = \nu = 1$, then

$$d_c(\Gamma\xi,\Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(3) Let $\xi = \nu = 2$, then

$$d_c(\Gamma\xi,\Gamma\nu) = 0 \leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]$$

(4) Let$\xi = 0, \nu = 1$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 0,\Gamma 1) = d_c(2,1) = \frac{3}{20}\\
&\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|\left(\frac{11}{20}\right) - (0)\right|\right]\\
&= \frac{2}{3}[d_c(0,1) + |d_c(0,2) - d_c(1,1)|]\\
&= \gamma[d_c(0,1) + |d_c(0,\Gamma 0) - d_c(1,\Gamma 1)|]\\
&= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

(5) Let $\xi = 1, \nu = 0$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 1,\Gamma 0) = d_c(1,2) = \frac{3}{20}\\
&\leq \frac{2}{3}\left[\left(\frac{1}{2}\right) + \left|(0) - \left(\frac{11}{20}\right)\right|\right]\\
&= \delta[d_c(1,0) + |d_c(1,1) - d_c(0,2)|]\\
&= \delta[d_c(1,0) + |d_c(1,\Gamma 1) - d_c(0,\Gamma 0)|]\\
&= \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

(6) Let $\xi = 0, \nu = 2$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 0,\Gamma 2) = d_c(2,1) = \frac{3}{20}\\
&\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

(7) Let$\xi = 2, \nu = 0$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 2,\Gamma 0) = d_c(1,2) = \frac{3}{20}\\
&\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

(8) Let $\xi = 1, \nu = 2$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 1,\Gamma 2) = d_c(1,1) = 0\\
&\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

(9) Let $\xi = 2, \nu = 1$, then

$$\begin{aligned}
&d_c(\Gamma\xi,\Gamma\nu) = d_c(\Gamma 2,\Gamma 1) = d_c(1,1) = 0\\
&\leq \gamma[d_c(\xi,\nu) + |d_c(\xi,\Gamma\xi) - d_c(\nu,\Gamma\nu)|]
\end{aligned}$$

Clearly, (2.2) is satisfied. On the other hand, note that (2.1) holds for all $\xi, \nu \in Y$. All other hypotheses of Theorem 2.1 are verified, and so Γ has a unique fixed point, which is $\xi^* = 1$.

4 Conclusion

It is concluded that a fixed-point theorem for E-contractions is required context of complete controlled metric spaces. We also provide examples to highlight the applicability of the findings made in E-contractive circumstances.

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Competing Interests

Authors have declared that no competing interests exist.

References

[1] Fulga A, Proca A. A new generalization of wardowski fixed point theorem in complete metric spaces. Adv. theory nonlinear anal. Its Appl. 2017;1:57-63.

[2] Alqahtani B, Fulga A, Karapınar E. A short note on the common fixed points of the Geraghty contraction of type ES,T. Demonstr Math. 2018;51(1):233-40.
DOI: 10.1515/dema-2018-0019

[3] Fulga A, Karapınar E. Revisiting of some outstanding metric fixed point theorems via E-contraction. Analele Univ. OvidiusConstanta-ser. Mat. 2018;26:73-97.

[4] Karapınar E, Fulga A, Aydi H. Study on Pata E-contractions. Adv Differ Equ. 2020;2020(1):539.
DOI: 10.1186/s13662-020-02992-4

[5] Dass BK, Gupta S. An extension of Banach contraction principle through rational expressions. Indian J Pure Appl Math. 1975;6:1455-8.

[6] Nazam M, Aydi H, Arshad M. A real generalization of the Dass-Gupta fixed point theorem. TWMS J Pure Appl Math. 2020;11(1):109-18.

[7] Czerwik S. Contraction mappings in b-metric spaces. Acta Math Inform Univ Ostraviensis. 1993;1:5-11.

[8] Afshari H, Aydi H, Karapınar E. On generalized α-ψ-Geraghty contractions on b-metric spaces. Georgian Math J. 2020;27(1):9-21.
DOI: 10.1515/gmj-2017-0063

[9] Ameer E, Aydi H, Arshad M, De la Sen M. Hybrid ćirić type graphic -contraction mappings with applications to electric circuit and fractional differential equations. Symmetry. 2020;12(3):467.
DOI: 10.3390/sym12030467

[10] Azam A, Mehmood N, Ahmad J, Radenović S. Multivalued fixed-point theorems in cone b-metric spaces. J Ineq Appl. 2013;2013(1). doi: 10.1186/1029-242X-2013-582.

[11] Huang H, Radenović S. Some fixed-point results of generalized lipschitz mappings on cone b-metric spaces over Banach algebras. J Comp Anal Appl. 2016;20:566-83.

[12] Karapinar E, Czerwik S, Aydi H(α,ψ)-Meir-Keeler contraction mappings in generalized-metric spaces," Journal of Function Spaces. 2018;2018:Article ID 3264620:4.

[13] Qawaqneh H, Md Noorani MMd, Shatanawi W, Aydi H, Alsamir H. Fixed point results for multi-valued contractions in b-metric spaces and an application. Mathematics. 2019;7(2):132.
DOI: 10.3390/math7020132

[14] Kamran T, Samreen M, UL Ain Q. A generalization of b-metric space and some fixed-point theorems. Mathematics. 2017;5(2):19.
DOI: 10.3390/math5020019

[15] Mlaiki N, Aydi H, Souayah N, Abdeljawad T. Controlled metric type spaces and the related contraction principle. Mathematics. 2018;6(10):194.
DOI: 10.3390/math6100194

[16] Abdeljawad T, Mlaiki N, Aydi H, Souayah N. Double controlled metric type spaces and some fixed-point results. Mathematics. 2018;6(12):320.
DOI: 10.3390/math6120320

[17] Wardowski D. Fixed points of a new type of contractive mappings in complete metric spaces. Fixed Point Theor Appl. 2012;2012(1).
DOI: 10.1186/1687-1812-2012-94

[18] Ahmad J, Al-Rawashdeh A, Azam A. New fixed point theorems for generalized F-contractions in complete metric spaces. Fixed Point Theor Appl. 2015;2015(1).
DOI: 10.1186/s13663-015-0333-2

[19] Ameer E, Aydi H, Arshad M, Alsamir H, Noorani M. Hybrid multivalued type contraction mappings in -complete partial b-metric spaces and applications. Symmetry. 2019;11(1):86.
DOI: 10.3390/sym11010086

[20] Budhia L, Aydi H, Ansari AH, Gopal D. Some new fixed point results in rectangular metric spaces with an application to fractional-order functional differential equations. Nonlinear Anal Modell Control. 2020;25(4):580-97.
DOI: 10.15388/namc.2020.25.17928

[21] Hussain N, Ahmad J, Azam A. On Suzuki-Wardowski type fixed point theorems. J Nonlinear Sci Appl. 2015;08(6):1095-111.
DOI: 10.22436/jnsa.008.06.19

[22] Hussain N, Ahmad J, Ćirić L, Azam A. Coincidence point theorems for generalized contractions with application to integral equations. Fixed Point Theor Appl. 2015;2015(1).
DOI: 10.1186/s13663-015-0331-4

[23] Ma Z, Asif A, Aydi H, Khan SU, Arshad M. Analysis of F-contractions in function weighted metric spaces with an application. Open Math. 2020;18(1):582-94.
DOI: 10.1515/math-2020-0035

[24] Olgun M, Alyıldız T, Biçer Ö, Altun I. Fixed point results for F-contractions on space with two metrics. Filomat. 2017;31(17):5421-6.
DOI: 10.2298/FIL1717421O

[25] Parvaneh V, Haddadi MR, Aydi H. On best proximity point results for some type of mappings. J Funct Spaces. 2020;2020:Article ID 6298138.
DOI: 10.1155/2020/6298138

[26] P. Patle, D. Patel, H. Aydi, and S. Radenovi Ć. On Type multivalued contractions and applications in symmetric and probabilistic spaces. Mathematics. 2019;7(2):144.

[27] Sahin H, Altun I, Turkoglu D. Two fixed point results for multivalued F-contractions on M-metric spaces. RACSAM. 2019;113(3):1839-49.
DOI: 10.1007/s13398-018-0585-x

[28] Shatanawi W, Karapinar E, Aydi H, Fulga A. 'Wardowski type contractions with applications on Caputo type nonlinear fractional differential equations' University Politechnica of Bucharest Scientific Bulletin-Series A-Applied,. 2020;82(2):157-70.

[29] Reich S. Fixed point of contractive functions. Boll Unione Mat Ital. 1972;4:26-42.

[30] Reich S. Kannan's fixed point theorem. Boll Unione Mat Ital. 1971;4:1-11.

[31] Samet B, Vetro C, Vetro P. Fixed point theorems for α−ψ-contractive type mappings. Nonlinear Anal Theor Methods Appl. 2012;75(4):2154-65.
DOI: 10.1016/j.na.2011.10.014

[32] Ahmad J, Eqal Al-Mazrooei AE, Aydi H, De la Sen M. On fixed point results in controlled metric spaces. J Funct Spaces. 2020;2020:Article ID 2108167:7.
DOI: 10.1155/2020/2108167

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