Hindawi
Advances in Mathematical Physics
Volume 2022, Article ID 8062119, 20 pages
https://doi.org/10.1155/2022/8062119

Research Article

Exact Solutions of the Nonlinear Space-Time Fractional Partial Differential Symmetric Regularized Long Wave (SRLW) Equation by Employing Two Methods

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Received 3 January 2022; Revised 11 February 2022; Accepted 24 March 2022; Published 9 May 2022

Academic Editor: Wen-Xiu Ma

In this article, with the aid of Maple software, the exact solutions to the space-time fractional symmetric regularized long wave (SRLW) equation are successfully examined by (G'/G^2)-expansion and extended complex methods. Consequently, three types of traveling wave solutions are found such as Weierstrass double periodic elliptic functions, simply periodic functions, and the rational function solutions. The obtained results will play an important role in understanding and studying SRLW equation. It is easy to see that the extended complex and (G'/G^2)-expansion methods are reliable and will be used extensively to seek exact solutions of any other fractional nonlinear partial differential equations (FNPDE).

1. Introduction

Fractional calculus is a 300-year-old mathematical problem. Despite its long history, its research has focused on the purely theoretical field of mathematics. However, in the recent decades, with the continuous expansion of fractional calculus applications, such as having memory and genetic characteristics, rheology, material and mechanical systems, electrical engineering, electromagnetism, signal processing and system identification, ANN (neural networks), and fractal and chaos, fractional partial differential equations have been developed into effective methods to unfold a series of strange events and processes [1]. Considerable physical phenomena are well modelled as FNPDE such as acoustic waves, acoustic gravity waves, hydromagnetic waves, fluid flow, chemistry, and other areas [2–6]. Due to the fast rapid development of computer computing power and with the help of symbolic computation software like Maple, Mathematica, and Matlab, the study of explicit solutions about FNPDE is deeply studied by both mathematicians and physicists. A great number of analytical and numerical solutions have been well established and applied to solve these FNPDE. Here are many methods as follows: homotopy analysis [7, 8], fractional (G'/G^2)-expansion [9, 10], tanh-function [11–13], extended tanh [14, 15], the fractional sub-equation [16–18], first integral [19, 20], functional variable [21], modified trial equation [22], finite-difference [23], and so on. In 2021, W. X. Ma et al. systematically studied N-soliton solutions to integrable equations with the help of Hirota direct method for both (1+1)-dimensional and (2+1)-dimensional integrable equations [24–26]. These have greatly promoted the study of nonlinear wave phenomena.

In 1984, inspired by weak nonlinear ionic acoustic and spatially charged wave models, Seyler and Fenstermacher [27] summed up the space-time fractional symmetric regularized long wave (SRLW) equation. This equation can summarize many physical phenomena, for instance: ion-acoustic waves in plasma and solitary waves with shallow water waves, shallow water waves. Since these equations are important in physics interpretation, a great number of ascendant and powerful methods have been proposed to obtain exact solutions of SRLW. In 2014, O. Gunerl and D. Eser applied functional variable, exp-function, and (G'/G^2)-expansion three methods to obtain the exact solutions of SRLW in the sense of the modified Riemann-Liouville derivative [28]. In 2015, M. Shakeel and S. T.

Mohyud-Din [29] used the fractional novel (G'/G^2)-expansion method to look for the exact solutions of SRLW and obtained many exact analytical solutions like hyperbolic function solutions, trigonometric function solutions, and rational solutions. In 2018, O. Guner and A. Bekir [30] presented exact analytical solutions of SRLW equation by using solitay wave ansatz method. In 2019, D. Yaroa et al. [31] used fractional complex transform and the revised Riemann-Liouville derivative to change the SRLW equation into the ordinary differential equations, and applied the improved F-expansion method to obtain the exact solutions which include hyperbolic and trigonometric solutions. In 2021, M. A. Khan, M. A. Akbar, and N. A. Hamid [32] applied the new auxiliary method to solve for the SRLW. In 2021, N. Maarouf et al. [33] investigated the Lie group analysis method of the SRLW equation and obtained the vector fields and similarity reductions of the equation. It showed that we can use a new independent variable to transform the governing FNPDE into a fractional nonlinear ordinary differential equation (FNODE). In 2021, N. Maarouf et al. [33] also obtained the exact solutions by using the power series expansion method. In 2021, S. C. Ünal et al. [34] got the exact solutions of SRLW equation inspired by a direct method based on the Jacobi elliptic functions ideas; furthermore, S. C. Ünal et al. [34] also obtained some general form solutions which include rational, trigonometric, and hyperbolic functions.

In our paper, the conformable fractional derivative of a function $f : [0,\infty) \longrightarrow R$ of order α is defined as

$$D_z^\alpha(f)(z) = \lim_{\varepsilon \longrightarrow 0} \frac{f\left(z + \varepsilon z^{1-\alpha}\right) - f(z)}{\varepsilon}, z > 0, \alpha \in (0, 1]. \quad (1)$$

If the above limitations exist, then f is called α-differentiable. Let $\alpha \in (0, 1]$ and f, g be α at point $z > 0$ differentiable, then D^α satisfies the following properties:

$$\begin{gathered}
(1) D^\alpha(af + bg) = aD^\alpha(f) + bD^\alpha(g), \text{for every } a, b \in R \\
(2) D^\alpha(z^n) = nz^{n-\alpha}, \text{for all } n \in R \\
(3) D^\alpha(\lambda) = 0, \text{then the constant function } f(z) = \lambda \\
(4) D^\alpha(fg) = fD^\alpha(g) + gD^\alpha(f), \\
(5) D^\alpha(f/g) = \{gD^\alpha(f) - fD^\alpha(g)\}/g^2, \\
(6) \text{If, in addition, } f \text{ is differentiable, then } D^\alpha(f)(z) = z^{1-\alpha}\frac{df}{dz}(z).
\end{gathered} \quad (2)$$

The nonlinear SRLW equation appears in several physical applications containing ion sound waves in plasma. This equation is an interesting model to describe ion-acoustic and space change waves and the real-valued $u(x, t)$ with weak nonlinearity. Now, let us study the SRLW equation ([35, 36])

$$D_t^{2\alpha}u + D_x^{2\alpha}u + uD_t^\alpha(D_x^\alpha u) + D_x^\alpha u D_t^\alpha u + D_t^{2\alpha}\left(D_x^{2\alpha}u\right) = 0, 0 < \alpha \leq 1. \quad (3)$$

Using the fractional transformation

$$u(x, t) = u(\xi), \xi = \frac{kx^\alpha}{\Gamma(1+\alpha)} + \frac{ct^\alpha}{\Gamma(1+\alpha)}, \quad (4)$$

here k, c are constants, then equation (7) turns into

$$c^2u'' + k^2u'' + ckuu'' + ck\left(u'\right)^2 + c^2k^2u^{iv} = 0. \quad (5)$$

Integration (4) twice yields

$$c^2k^2u'' + \left(c^2 + k^2 + r\right)u + \frac{1}{2}cku^2 + s = 0, \quad (6)$$

here r, s are integral constants.

In order to obtain the exact solution of SRLW equation, the extended complex method is employed by us to seek the exact solutions of equation (6), and another way we use the (G'/G^2)-expansion method to get some exact solutions of SRLW equation.

Considering the following form of a NFPDE

$$P\left(u, u_x, u_t, u_{xt}, u_x^\alpha, u_t^\alpha, u_{xx}^{2\alpha}, u_{tt}^{2\alpha}, u_{xt}^{2\alpha} \cdots\right) = 0, 0 < \alpha \leq 1. \quad (7)$$

in equation (7), P is a polynomial with an unknown function $\mu(x, t)$ and its fractional derivatives, involving non-nonlinear terms and the highest order derivatives.

In order to verify the accuracy of the obtained results, we define accuracy analysis of error function (AAEF) as follows

$$AAEF = c^2k^2u'' + \left(c^2 + k^2 + r\right)u + \frac{1}{2}cku^2 + s. \quad (8)$$

It should be noted that the smaller the AAEF, the closer the approximate solution is to the exact solution. When the error infinitely approaches zero, we consider the result to be closer to the true value. Therefore, we use AAEF to indicate the accuracy of the results.

The structure of this article is as follows: in Section 2, we probe the modified (G'/G^2)-expansion method to the SRLW equation; in Section 3, we discuss the extended complex method and main result; in Sections 4 and 5, we give the detail proof of the main results in this paper; in Section 6, we give the specific form of accuracy analysis of error function to show the accuracy of our results; in Section 7, we compared our methods and results with others in detail; in Section 8, through computer simulation images, we further analyze the nature of the solutions that we have obtained; finally, the conclusion and future recommendations in Section 9.

2. Employing of the Modified (G'/G^2)-Expansion Method to the SRLW Equation

Next, we will use the recently established modified (G'/G^2)-expansion method [37, 38] to provide new and more general traveling wave solutions for the above equation (6).

Step 1. Inserting the traveling wave transform $T : u(x, t) = u(\xi), \xi = (kx^\alpha/\Gamma(1+\alpha)) + (ct^\alpha/\Gamma(1+\alpha))$ into equation

(7), changing it to the following integer order ordinary differential equation (IOODE):

$$K\left(u, u', u'', u''',\cdots\right) = 0, \tag{9}$$

in the above equation (9), K is a polynomial composed of $u(\xi)$ and its integer order derivatives.

Step 2. Regarding that the solution of equation (9) can be expressed by a polynomial in (G'/G^2) as follows:

$$u(\xi) = \alpha_0 + \sum_{n=1}^{m}\left(\alpha_m\left(\frac{G'}{G^2}\right)^m + \beta_m\left(\frac{G'}{G^2}\right)^{-m}\right), \tag{10}$$

in equation (10), $G = G(\xi)$ satisfies the following differential equation as follows:

$$\left(\frac{G'}{G^2}\right)' = \sigma + \mu\left(\frac{G'}{G^2}\right) + \rho\left(\frac{G'}{G^2}\right)^2, \tag{11}$$

where μ, σ, and ρ are free constants. The positive integer m can be determined by considering the uniform equilibrium between the highest order derivatives and the nonlinear term appearing in the ODE (8).

Step 3. Taking equation (10) into equation (9), and using equation (11), the left side (8) of the formula was converted to another polynomial in the (G'/G^2). Computing all the coefficients of the polynomial to zero yields the algebraic equations for $\alpha_m, \cdots, \lambda$, and μ.

Step 4. The undetermined constants$\alpha_m,\cdots,\alpha_0,\beta_1,\cdots,\beta_m$ can be obtained by solving the system of algebraic equations obtained in Step 3, since the general form solutions of (10) have five possible solutions as following. Hence, the exact solutions of the given Eq. (7) can be obtained as follows.

Here:

$$\begin{cases} \text{Case1 : If } \sigma\rho > 0, \mu = 0, \text{ then} \\ \left(\dfrac{G'}{G^2}\right)(\xi) = \dfrac{\sqrt{\sigma\rho}}{\sigma}\left[\dfrac{C_1 \cos\sqrt{\sigma\rho}\xi + C_2 \sin\sqrt{\sigma\rho}\xi}{C_2 \cos\sqrt{\sigma\rho}\xi - C_1 \sin\sqrt{\sigma\rho}\xi}\right]; \\ \text{Case2 : If } \sigma\rho < 0, \mu = 0, \text{ then} \\ \left(\dfrac{G'}{G^2}\right)(\xi) = -\dfrac{\sqrt{|\sigma\rho|}}{\sigma}\left[\dfrac{C_1 \sinh 2\sqrt{|\sigma\rho|}\xi + C_2 \cosh 2\sqrt{|\sigma\rho|}\xi + C_2}{C_1 \cosh 2\sqrt{|\sigma\rho|}\xi + C_1 \sinh 2\sqrt{|\sigma\rho|}\xi - C_2}\right]; \\ \text{Case3 : If } \sigma = 0, \rho \neq 0, \mu = 0, \text{ then} \\ \left(\dfrac{G'}{G^2}\right)(\xi) = -\dfrac{C_1}{\rho(C_1\xi + C_2)}; \\ \text{Case4 : If } \mu \neq 0, \Delta \geq 0, \text{ then} \\ \left(\dfrac{G'}{G^2}\right)(\xi) = -\dfrac{\mu}{2\rho} - \left[\dfrac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right]; \\ \text{Case5 : If } \mu \neq 0, \Delta < 0, \text{ then} \\ \left(\dfrac{G'}{G^2}\right)(\xi) = -\dfrac{\mu}{2\rho} - \left[\dfrac{\sqrt{-\Delta}\left(C_1 \cos\left(\sqrt{-\Delta}/2\right)\xi - C_2 \sin\left(\sqrt{-\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cos\left(\sqrt{-\Delta}/2\right)\xi + C_1 \sin\left(\sqrt{-\Delta}/2\right)\xi\right)}\right]; \end{cases} \tag{12}$$

where C_1, C_2 are arbitrary constants and $\Delta = \mu^2 - 4\rho\sigma$.

Step 5. Putting the inverse transform T^{-1} into the solutions $u(\xi)(\xi = (kx^\alpha/\Gamma(1+\alpha)) + (ct^\alpha/\Gamma(1+\alpha)))$, we can get all exact solutions $u(x, t)$ of the original FNPDE.

$$\begin{aligned} u_{11}(\xi) = {}& -\frac{c^2 + k^2 + r}{ck} \pm 8\sigma\rho k - 12\frac{\rho^3 ck}{\sigma} \\ & \cdot \left[\frac{C_1 \cos\sqrt{\sigma\rho}\xi + C_2 \sin\sqrt{\sigma\rho}\xi}{C_1 \sin\sqrt{\sigma\rho}\xi - C_2 \cos\sqrt{\sigma\rho}\xi}\right]^2 \\ & - 12\frac{\sigma^3 ck}{\rho}\left[\frac{C_1 \cos\sqrt{\sigma\rho}\xi + C_2 \sin\sqrt{\sigma\rho}\xi}{C_1 \sin\sqrt{\sigma\rho}\xi - C_2 \cos\sqrt{\sigma\rho}\xi}\right]^{-2}, \end{aligned} \tag{13}$$

Remark 1. From the above five cases, (11) contains trigonometric (Cases 1 and 5), rational (Case 3), and hyperbolic (Cases 2 and 4) three forms solutions.

Theorem 2. *By (G'/G^2)-expansion method, we have found the following five cases solutions of Eq. (6):*

Case 1. If $\sigma\rho > 0$, $\mu = 0$, then

and $(c^2 + k^2 + r)^2 - 2cks = 256\sigma^2\rho^2c^2k^4$.

$$\begin{aligned} u_{12}(\xi) = {}& -\frac{c^2 + k^2 + r}{ck} \pm 8\sigma\rho k + 12\frac{\rho^3 ck}{\sigma} \\ & \cdot \left[\frac{C_1 \sinh 2\sqrt{-\sigma\rho}\xi + C_2 \cosh 2\sqrt{-\sigma\rho}\xi + C_2}{C_1 \cosh 2\sqrt{-\sigma\rho}\xi + C_1 \sinh 2\sqrt{-\sigma\rho}\xi - C_2}\right]^2 \\ & + 12\frac{\sigma^3 ck}{\rho}\left[\frac{C_1 \sinh 2\sqrt{-\sigma\rho}\xi + C_2 \cosh 2\sqrt{-\sigma\rho}\xi + C_2}{C_1 \cosh 2\sqrt{-\sigma\rho}\xi + C_1 \sinh 2\sqrt{-\sigma\rho}\xi - C_2}\right]^{-2}, \end{aligned} \tag{14}$$

Case 2. If $\sigma\rho < 0$, $\mu = 0$, then

and $(c^2 + k^2 + r)^2 - 2cks = 256\sigma^2\rho^2c^2k^4$.

$$u_{13}(\xi) = -\frac{c^2 + k^2 + r}{ck} - \frac{12ckC_1^2}{(C_1\xi + C_2)^2} \tag{15}$$

Case 3. If $\sigma = 0$ and $\rho \neq 0$, $\mu = 0$, then

and $(c^2 + k^2 + r)^2 = 2cks$.

$$\begin{aligned} u_{14}(\xi) = {}& 2\mu^2 ck - \frac{c^2 + k^2 + r}{ck} \\ & - 3\mu^2 ck\left(\frac{C_1 \cosh(\pm\mu/2)\xi + C_2 \sinh(\pm\mu/2)\xi}{C_2 \cosh(\pm\mu/2)\xi + C_1 \sinh(\pm\mu/2)\xi}\right)^2 \end{aligned} \tag{16}$$

Case 4. If $\mu \neq 0$ and $\Delta = \mu^2 - 4\sigma\rho \geq 0$, then and $(c^2+k^2+r)^2-2cks=u^4c^2k^2$, $\sigma=0$ and $\rho\neq 0$.

$$u_{15}(\xi)=2\mu^2ck-\frac{c^2+k^2+r}{ck}+3\mu^2ck\left(\frac{C_1\cos(\pm i\mu/2)\xi-C_2\sin(\pm i\mu/2)\xi}{C_2\cos(\pm i\mu/2)\xi+C_1\sin(\pm i\mu/2)\xi}\right)^2 \tag{17}$$

Case 5. If $\mu\neq 0$ and $\Delta=\mu^2-4\sigma\rho<0$, then

and $(c^2+k^2+r)^2-2cks=u^4c^2k^2$, $\sigma=0$ and $\rho\neq 0$.

Here $\xi=(kx^\alpha/\Gamma(1+\alpha))+(ct^\alpha/\Gamma(1+\alpha))$. C_1 and C_2 are arbitrary constants.

3. Introduction of the Extended Complex Method and Main Result

The extended complex number method involves the knowledge related to the Weierstrass elliptic function. First, we give the brief introduction of Weierstrass elliptic function: $\wp(\xi):=\wp(\xi,g_2,g_3)$ is a meromorphic function in the complex plane $\mathbb{C}$ with double periods ω_1,ω_2 [39–41] and defined as

$$\wp(\xi;w_1,w_2):=\frac{1}{\xi^2}+\sum_{\mu,\nu\in Z,\mu^2+\nu^2\neq 0}\left\{\frac{1}{(\xi+\mu w_1+\nu w_2)^2}-\frac{1}{(\mu w_1+\nu w_2)^2}\right\}, \tag{18}$$

which satisfies the following equation

$$\left(\wp'(\xi)\right)^2=4\wp(\xi)^3-g_2\wp(\xi)-g_3, \tag{19}$$

in equation (19), $g_2=60s_4$, $g_3=140s_6$, and $\Delta(g_2,g_3)\neq 0$, and has the another formula

$$\wp(\xi-\xi_0)=-\wp(\xi)-\wp(\xi_0)+\frac{1}{4}\left[\frac{\wp'(\xi)+\wp'(\xi_0)}{\wp(\xi)-\wp(\xi_0)}\right]^2. \tag{20}$$

Next, we will employ the recently established extended complex method [42–45] to provide new and more general traveling wave solutions to the equations mentioned above.

Step 1. Inserting the traveling wave transform T: $u(x,t)=u(\xi)$, $\xi=(kx^\alpha/\Gamma(1+\alpha))+(ct^\alpha/\Gamma(1+\alpha))$ into equation (7), alternating it to the following (IOODE):

$$K\left(u,u',u'',u''',\cdots\right)=0, \tag{21}$$

here K is a polynomial of $u(\xi)$ and its derivatives.

Step 2. Next, we will find out weak $\langle p,q\rangle$ condition. To find out the weak $\langle p,q\rangle$ condition of equation (21), the Laurent series,

$$u(\xi)=\sum_{k=-q}^{\infty}c_k\xi^k,\ q>0,\ c_{-q}\neq 0, \tag{22}$$

are replaced into equation (21), then the p distinct Laurent singular parts are obtained as below:

$$\sum_{k=-q}^{-1}c_k\xi^k. \tag{23}$$

Here p indicates that there are p distinct meromorphic solutions in the equation, and q means that their poles at $\xi=0$ have q multiple roots in the equation.

Step 3. Take the following pending forms

$$u(\xi)=\sum_{i=1}^{l-1}\sum_{j=2}^{q_i}\frac{(-1)^jc_{-ij}}{(j-1)!}\frac{d^{j-2}}{dz^{j-2}}\left(\frac{1}{4}\left[\frac{\wp'(\xi)+B_i}{\wp(\xi)-A_i}\right]^2-\wp(\xi)\right)+\sum_{i=1}^{l-1}\frac{c_{-i1}}{2}\frac{\wp'(\xi)+B_i}{\wp(\xi)-A_i}+\sum_{j=2}^{q_l}\frac{(-1)^jc_{-lj}}{(j-1)!}\frac{d^{j-2}}{dz^{j-2}}\wp(\xi)+c_0, \tag{24}$$

$$u(\xi)=\sum_{i=1}^{l}\sum_{j=1}^{q}\frac{c_{ij}}{(\xi-\xi_i)^j}+c_0, \tag{25}$$

$$u(\zeta)=\sum_{i=1}^{l}\sum_{j=1}^{q}\frac{c_{ij}}{(\zeta-\zeta_i)^j}+c_0,\ \text{here}\ \zeta=e^{\vartheta\xi}\ (\vartheta\in\mathbb{C}). \tag{26}$$

In equation (24), c_{-ij} are given as shown in equation (21), and $B_i^2=4A_i^3-g_2A_i-g_3$, $\Sigma_{i=1}^{l}c_{-i1}=0$, and equations (23), (24), and (25) have $l(\leq p)$ distinct poles of multiplicity q.

Step 4. We can get meromorphic solutions and the above addition formulas. Putting the inverse transform T^{-1} into the solutions $u(\xi)$, we can obtain all exact solutions $u(x,t)$ of the original FNPDE.

(a) The rational function solutions

$$u_{21}(\xi)=-\frac{12ck}{(\xi-\xi_0)^2}-\frac{c^2+k^2+r}{ck}, \tag{27}$$

where $2cks=(c^2+k^2+r)^2$, $\xi_0\in\mathbb{C}$.

(b) The simply periodic solutions are obtained by

$$u_{22}(\xi)=-12ck\vartheta^2\coth^2\frac{\vartheta}{2}(\xi-\xi_0)-ck\vartheta^2-\frac{c^2+k^2+r}{ck}, \tag{28}$$

where $2cks=-c^4k^4\vartheta^4+(c^2+k^2+r)^2$, $\xi_0\in\mathbb{C}$, $\vartheta\in\mathbb{C}$.

(c) The Weierstrass elliptic solutions are as follows:

$$u_{23}(\xi)=-12ck\left\{-\wp(\xi)+\frac{1}{4}\left[\frac{\wp'(\xi)+F}{\wp(\xi)-E}\right]^2\right\}+12ckE-\frac{c^2+k^2+r^2}{ck}, \tag{29}$$

Table 1: AAEF ($C_1 \neq 0$, $C_2 = 0$) for the solution $u_{11}(\xi)$ in eq. (13).

α	c	k	ρ	σ	x	t	REF
0.25	0.001	0.001	$\frac{\pi}{4}$	$\frac{\pi}{4}$	196249	196249	$-6.39625984 * 10^{-11}$
0.5	0.001	0.001	$\frac{\pi}{4}$	$\frac{\pi}{4}$	196249	196249	$-3.660149852 * 10^{-11}$
0.75	0.001	0.001	$\frac{\pi}{4}$	$\frac{\pi}{4}$	196249	196249	$-3.720239068 * 10^{-11}$
0.25	0.001	0.001	$\frac{\pi}{4}$	$\frac{\pi}{4}$	150000	150000	$-6.79117167 * 10^{-11}$
0.5	0.001	0.001	$\frac{\pi}{4}$	$\frac{\pi}{4}$	150000	150000	$-3.728142221 * 10^{-11}$
0.75	0.001	0.001	$\pi/4$	$\pi/4$	150000	150000	$-3.668526651 * 10^{-11}$
0.25	0.001	0.001	$\pi/4$	$\pi/4$	250000	250000	$-6.083233454 * 10^{-11}$
0.5	0.001	0.001	$\pi/4$	$\pi/4$	250000	250000	$-3.660763965 * 10^{-11}$
0.75	0.001	0.001	$\pi/4$	$\pi/4$	250000	250000	$-3.704605275 * 10^{-11}$

Table 2: AAEF ($C_1 = 0$, $C_2 \neq 0$) for the solution $u_{12}(\xi)$ in eq. (14).

α	c	k	ρ	σ	x	t	REF
0.25	0.001	0.001	2	−2	196249	196249	$-1.541853209 * 10^{-9}$
0.5	0.001	0.001	2	−2	196249	196249	$-2.024770373 * 10^{-9}$
0.25	0.001	0.001	2	−2	150000	150000	$-1.541838967 * 10^{-9}$
0.5	0.001	0.001	2	−2	150000	150000	$-1.898189859 * 10^{-9}$
0.25	0.001	0.001	2	−2	250000	250000	$-1.541867807 * 10^{-9}$
0.5	0.001	0.001	2	−2	250000	250000	$5.931281180 * 10^{-9}$

here $2cks = -12c^4k^4g_2 + (c^2 + k^2 + r)^2$, $F^2 = 4E^3 - g_2 E - g_3$, g_3 and E are arbitrary constants.

Theorem 3. *Suppose $ck \neq 0$, by the extended complex method, we have found the following three cases solutions of Eq. (6):*

All in above, $\xi = (kx^\alpha/\Gamma(1+\alpha)) + (ct^\alpha/\Gamma(1+\alpha))$.

4. Proof of Theorem 2

Considering the homogeneous equilibrium term between u'' and u^2 in (5), we deduce $m = 2$. So we can infer the solution of (5) as follows:

$$u(\xi) = \alpha_2\left(\frac{G'}{G^2}\right)^2 + \alpha_1\left(\frac{G'}{G^2}\right) + \alpha_0 + \beta_2\left(\frac{G'}{G^2}\right)^{-2} + \beta_1\left(\frac{G'}{G^2}\right)^{-1}, \tag{30}$$

here $\alpha_0, \alpha_1, \alpha_2$ are the upcoming constants that will be determined later.

Next, we will use (29) and (10) to collect and sort out all terms with the same power of (G'/G^2) together.

First, from (29), we get

$$\begin{aligned} u'(\xi) &= (\alpha_1\sigma - \rho\beta_1) + (\alpha_1\mu + 2\sigma\alpha_2)\left(\frac{G'}{G^2}\right) \\ &\quad + (\rho\alpha_1 + 2\mu\alpha_2)\left(\frac{G'}{G^2}\right)^2 + 2\rho\alpha_2\left(\frac{G'}{G^2}\right)^3 \\ &\quad - (\beta_1\sigma + 2\beta_2\mu)\left(\frac{G'}{G^2}\right)^{-2} \\ &\quad - (\mu\beta_1 + 2\beta_2\mu)\left(\frac{G'}{G^2}\right)^{-1} - 2\beta_2\sigma\left(\frac{G'}{G^2}\right)^{-3}. \end{aligned} \tag{31}$$

Substituting (10) into (30), we obtain

$$\begin{aligned} u''(\xi) &= (\alpha_1\sigma\mu + 2\sigma^2\alpha_2 + \rho\mu\beta_1 + 2\beta_2\rho^2) \\ &\quad + (\alpha_1\mu^2 + 2\mu\sigma\alpha_2 + 2\rho\alpha_1\sigma + 2\mu\alpha_2\sigma)\left(\frac{G'}{G^2}\right) \\ &\quad + (\alpha_1\mu\rho + 2\sigma\alpha_2\rho + 2\rho\alpha_1\mu + 4\mu^2\alpha_2 + 6\rho\sigma\alpha_2)\left(\frac{G'}{G^2}\right)^2 \\ &\quad + (2\rho^2\alpha_1 + 4\mu\rho\alpha_2 + 6\rho\alpha_2\mu)\left(\frac{G'}{G^2}\right)^3 + 6\rho^2\alpha_2\left(\frac{G'}{G^2}\right)^4 \\ &\quad + (2\beta_1\sigma^2 + 4\beta_2\mu\sigma + 6\beta_2\sigma\mu)\left(\frac{G'}{G^2}\right)^{-3} \\ &\quad + (2\beta_1\sigma\mu + 4\beta_2\mu^2 + \mu\beta_1\sigma + 2\beta_2\rho\sigma + 6\rho\beta_2\sigma)\left(\frac{G'}{G^2}\right)^{-2} \\ &\quad + (2\rho\beta_1\sigma + 4\beta_2\rho\mu + \mu^2\beta_1 + 2\beta_2\mu\rho)\left(\frac{G'}{G^2}\right)^{-1} + 6\beta_2\sigma^2\left(\frac{G'}{G^2}\right)^{-4}. \end{aligned} \tag{32}$$

Now, we put (29), (30), and (31) into (5), and sort out all terms with the same power of $(G'(\xi)/G^2(\xi))$ together,

$$
\begin{aligned}
&\left(6\rho^2\alpha_2c^2k^2 + \frac{1}{2}\alpha_2^2ck\right)\left(\frac{G'(\xi)}{G^2(\xi)}\right)^4 \\
&\quad + \left(2\rho^2\alpha_1c^2k^2 + 10\mu\rho\alpha_2c^2k^2 + \alpha_1\alpha_2ck\right)\left(\frac{G'(\xi)}{G^2(\xi)}\right)^3 \\
&\quad + \left(3\rho\mu\alpha_1c^2k^2 + 8\sigma\alpha_2\rho c^2k^2 + 4\mu^2\alpha_2c^2k^2 + c^2\alpha_2\right. \\
&\quad \left. + k^2\alpha_2 + r\alpha_2 + \frac{1}{2}ck\alpha_1^2 + \alpha_0\alpha_2ck\right)\left(\frac{G'(\xi)}{G^2(\xi)}\right)^2 \\
&\quad + \left(\alpha_1\mu^2c^2k^2 + 4\mu\sigma\alpha_2c^2k^2 + 2\rho\alpha_1\sigma c^2k^2 + c^2\alpha_1 + k^2\alpha_1 + r\alpha_1 + \alpha_0\alpha_1ck + \alpha_2\beta_1ck\right) \\
&\quad \cdot \left(\frac{G'(\xi)}{G^2(\xi)}\right) + \left(c^2k^2\alpha_1\sigma\mu + 2\sigma^2\alpha_2c^2k^2 + \rho\mu\beta_1c^2k^2 + 2\beta_2\rho^2c^2k^2\right. \\
&\quad \left. + \frac{1}{2}ck\alpha_0^2 + \alpha_1\beta_1ck + \alpha_2\beta_2ck + c^2\alpha_0 + k^2\alpha_0 + r\alpha_0 + s\right) + \left(6c^2k^2\beta_2\sigma^2 + \frac{1}{2}ck\beta_2^2\right) \\
&\quad \cdot \left(\frac{G'(\xi)}{G^2(\xi)}\right)^{-4} + \left(2\beta_1\sigma^2c^2k^2 + 10\beta_2\mu\sigma c^2k^2 + \beta_1\beta_2ck\right)\left(\frac{G'(\xi)}{G^2(\xi)}\right)^{-3} \\
&\quad + \left(3\beta_1\sigma\mu c^2k^2 + 4\beta_2\mu^2c^2k^2 + 8\beta_2\rho\sigma c^2k^2 + c^2\beta_2 + k^2\beta_2 + r\beta_2 + \frac{1}{2}ck\beta_1^2 + \alpha_0\beta_2ck\right) \\
&\quad \cdot \left(\frac{G'(\xi)}{G^2(\xi)}\right)^{-2} + \left(2\rho\beta_1\sigma c^2k^2 + 6\beta_2\rho\mu c^2k^2 + \mu^2\beta_1c^2k^2 + c^2\beta_1\right. \\
&\quad \left. + k^2\beta_1 + r\beta_1 + \alpha_0\beta_1ck + \alpha_1\beta_2ck\right)\left(\frac{G'(\xi)}{G^2(\xi)}\right)^{-1} = 0.
\end{aligned} \tag{33}
$$

For the $(G'(\xi))/(G^2(\xi))$ functions of the same terms power, extracting its undetermined coefficients and set to zero, and the following equations can be obtained:

$$
\begin{cases}
c^2k^2\alpha_1\sigma\mu + 2\sigma^2\alpha_2c^2k^2 + \rho\mu\beta_1c^2k^2 + 2\beta_2\rho^2c^2k^2 + \frac{1}{2}ck\alpha_0^2 + \alpha_1\beta_1ck + \alpha_2\beta_2ck + c^2\alpha_0 + k^2\alpha_0 + r\alpha_0 + s = 0, (1.1) \\
\alpha_1\mu^2c^2k^2 + 4\mu\sigma\alpha_2c^2k^2 + 2\rho\alpha_1\sigma c^2k^2 + c^2\alpha_1 + k^2\alpha_1 + r\alpha_1 + \alpha_0\alpha_1ck + \alpha_2\beta_1ck = 0, (1.2) \\
3\rho\mu\alpha_1c^2k^2 + 8\sigma\alpha_2\rho c^2k^2 + 4\mu^2\alpha_2c^2k^2 + c^2\alpha_2 + k^2\alpha_2 + r\alpha_2 + \frac{1}{2}ck\alpha_1^2 + \alpha_0\alpha_2ck = 0, (1.3) \\
2\rho^2\alpha_1c^2k^2 + 10\mu\rho\alpha_2c^2k^2 + \alpha_1\alpha_2ck = 0, (1.4) \\
6\rho^2\alpha_2c^2k^2 + \frac{1}{2}\alpha_2^2ck = 0, (1.5) \\
6c^2k^2\beta_2\sigma^2 + \frac{1}{2}ck\beta_2^2 = 0, (1.6) \\
2\beta_1\sigma^2c^2k^2 + 10\beta_2\mu\sigma c^2k^2 + \beta_1\beta_2ck = 0, (1.7) \\
3\beta_1\sigma\mu c^2k^2 + 4\beta_2\mu^2c^2k^2 + 8\beta_2\rho\sigma c^2k^2 + c^2\beta_2 + k^2\beta_2 + r\beta_2 + \frac{1}{2}ck\beta_1^2 + \alpha_0\beta_2ck = 0, (1.8) \\
2\rho\beta_1\sigma c^2k^2 + 6\beta_2\rho\mu c^2k^2 + \mu^2\beta_1c^2k^2 + c^2\beta_1 + k^2\beta_1 + r\beta_1 + \alpha_0\beta_1ck + \alpha_1\beta_2ck = 0. (1.9).
\end{cases} \tag{34}
$$

Case 1. If $\sigma\rho > 0$, $\mu = 0$, under this assumption, solving the above system equation (33), we get

$$
\begin{cases}
\alpha_0 = -\dfrac{c^2 + k^2 + r}{ck} \pm 8\sigma\rho k, \\
\alpha_1 = 0, \\
\alpha_2 = -12\rho^2ck, \\
\beta_1 = 0, \\
\beta_2 = -12\sigma^2ck,
\end{cases} \tag{35}
$$

here

$$
\begin{gathered}
\left(c^2 + k^2 + r\right)^2 - 2cks = 256\sigma^2\rho^2c^2k^4 \\
\left(\frac{G'}{G^2}\right)(\xi) = \frac{\sqrt{\sigma\rho}}{\sigma}\left[\frac{C_1\cos\sqrt{\sigma\rho}\xi + C_2\sin\sqrt{\sigma\rho}\xi}{C_1\sin\sqrt{\sigma\rho}\xi - C_2\cos\sqrt{\sigma\rho}\xi}\right].
\end{gathered} \tag{36}
$$

Now, we give the forms of $u(\xi)$:

$$
\begin{aligned}
u_{11}(\xi) &= \alpha_0 + \alpha_2\left(\frac{G'}{G^2}\right)^2 + \beta_2\left(\frac{G'}{G^2}\right)^{-2} = -\frac{c^2 + k^2 + r}{ck} \\
&\quad \pm 8\sigma\rho k - 12\frac{\rho^3ck}{\sigma}\left[\frac{C_1\cos\sqrt{\sigma\rho}\xi + C_2\sin\sqrt{\sigma\rho}\xi}{C_1\sin\sqrt{\sigma\rho}\xi - C_2\cos\sqrt{\sigma\rho}\xi}\right]^2 \\
&\quad - 12\frac{\sigma^3ck}{\rho}\left[\frac{C_1\cos\sqrt{\sigma\rho}\xi + C_2\sin\sqrt{\sigma\rho}\xi}{C_1\sin\sqrt{\sigma\rho}\xi - C_2\cos\sqrt{\sigma\rho}\xi}\right]^{-2},
\end{aligned} \tag{37}
$$

and $\left(c^2 + k^2 + r\right)^2 - 2cks = 256\sigma^2\rho^2c^2k^4$.

Case 2. If $\sigma\rho < 0$, $\mu = 0$, under this assumption, solving the above system equation (33), we get

$$
\begin{cases}
\alpha_0 = -\dfrac{c^2 + k^2 + r}{ck} \pm 8\sigma\rho k, \\
\alpha_1 = 0, \\
\alpha_2 = -12\rho^2ck, \\
\beta_1 = 0, \\
\beta_2 = -12\sigma^2ck,
\end{cases} \tag{38}
$$

here

$$
\left(c^2 + k^2 + r\right)^2 - 2cks = 256\sigma^2\rho^2c^2k^4 \tag{39}
$$

and then the result is similar with Case 1.

$$
\left(\frac{G'}{G^2}\right)(\xi) = -\frac{\sqrt{|\sigma\rho|}}{\sigma}\left[\frac{C_1\sinh 2\sqrt{|\sigma\rho|}\xi + C_2\cosh 2\sqrt{|\sigma\rho|}\xi + C_2}{C_1\cosh 2\sqrt{|\sigma\rho|}\xi + C_1\sinh 2\sqrt{|\sigma\rho|}\xi - C_2}\right]. \tag{40}
$$

Now, we give the forms of $u(\xi)$:

$$
\begin{aligned}
u_{12}(\xi) &= \alpha_0 + \alpha_2\left(\frac{G'}{G^2}\right)^2 + \beta_2\left(\frac{G'}{G^2}\right)^{-2} = -\frac{c^2 + k^2 + r}{ck} \pm 8\sigma\rho k \\
&\quad + 12\frac{\rho^3ck}{\sigma}\left[\frac{C_1\sinh 2\sqrt{-\sigma\rho}\xi + C_2\cosh 2\sqrt{-\sigma\rho}\xi + C_2}{C_1\cosh 2\sqrt{-\sigma\rho}\xi + C_1\sinh 2\sqrt{-\sigma\rho}\xi - C_2}\right]^2 \\
&\quad + 12\frac{\sigma^3ck}{\rho}\left[\frac{C_1\sinh 2\sqrt{-\sigma\rho}\xi + C_2\cosh 2\sqrt{-\sigma\rho}\xi + C_2}{C_1\cosh 2\sqrt{-\sigma\rho}\xi + C_1\sinh 2\sqrt{-\sigma\rho}\xi - C_2}\right]^{-2},
\end{aligned} \tag{41}
$$

and $\left(c^2 + k^2 + r\right)^2 - 2cks = 256\sigma^2\rho^2c^2k^4$.

Table 3: AAEF for the solution $u_{22}(\xi)$ in eq. (63).

α	c	k	ξ_0	ϑ	x	t	REF
0.25	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	196249	196249	$1.714265321 * 10^{-18}$
0.5	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	196249	196249	$1.714265334 * 10^{-18}$
0.75	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	196249	196249	$1.714265297 * 10^{-18}$
0.25	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	150000	150000	$1.714265321 * 10^{-18}$
0.5	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	150000	150000	$1.714265334 * 10^{-18}$
0.75	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	150000	150000	$1.714265297 * 10^{-18}$
0.25	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	250000	250000	$1.714265321 * 10^{-18}$
0.5	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	250000	250000	$1.714265334 * 10^{-18}$
0.75	0.001	0.001	$\xi_0 = 1 - 20\sqrt{7}$	$1/\sqrt{7}$	250000	250000	$1.714265297 * 10^{-18}$

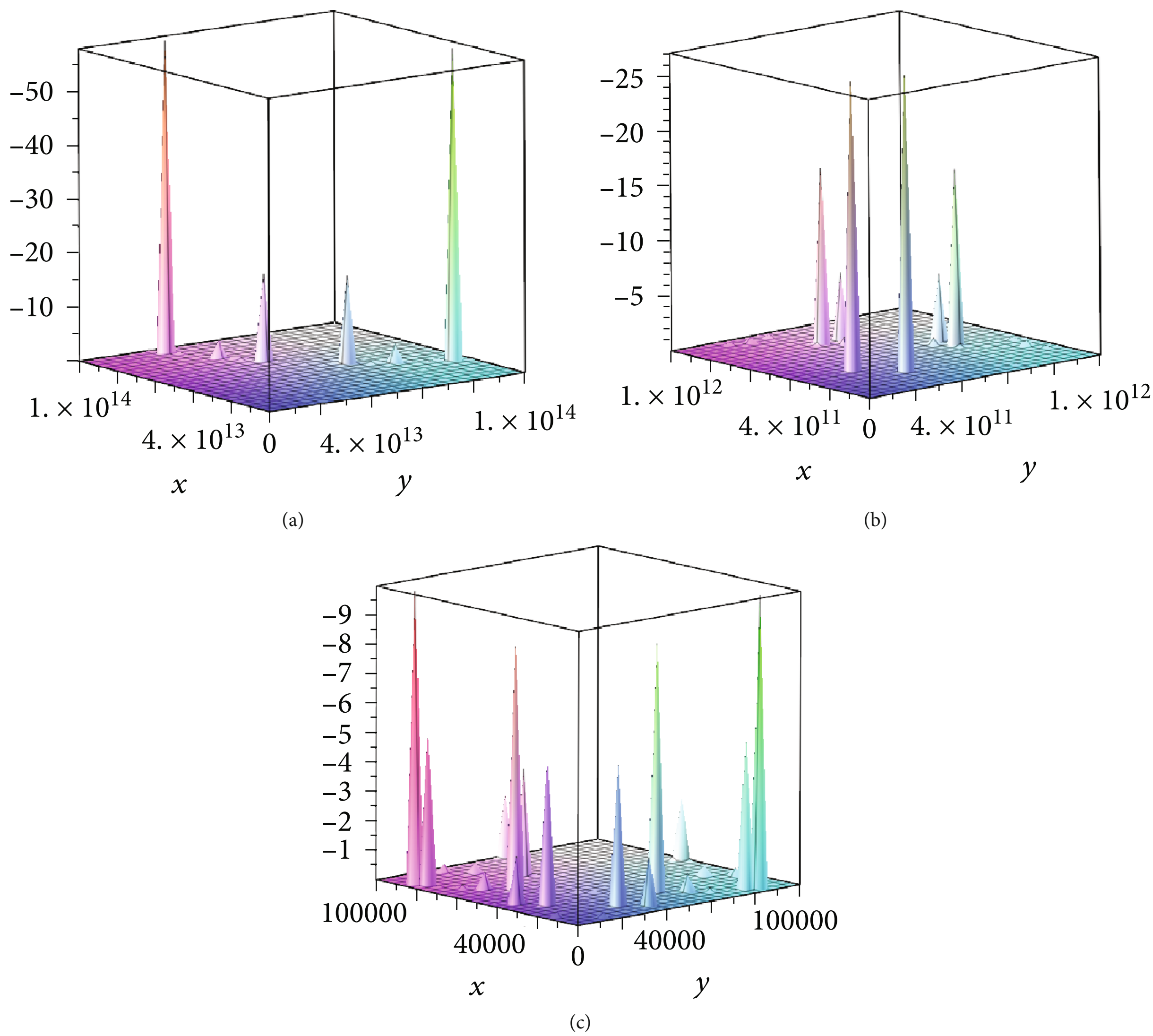

Figure 1: The three-dimensional images of $u_{11}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = \pi/4$, $\sigma = \pi/4$, $s = 0$, $C_2 = 0$; from (*a*) to (*c*), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. Three graphs demonstrate the multisoliton profiles of $u_{11}(\xi)$ on the domain.

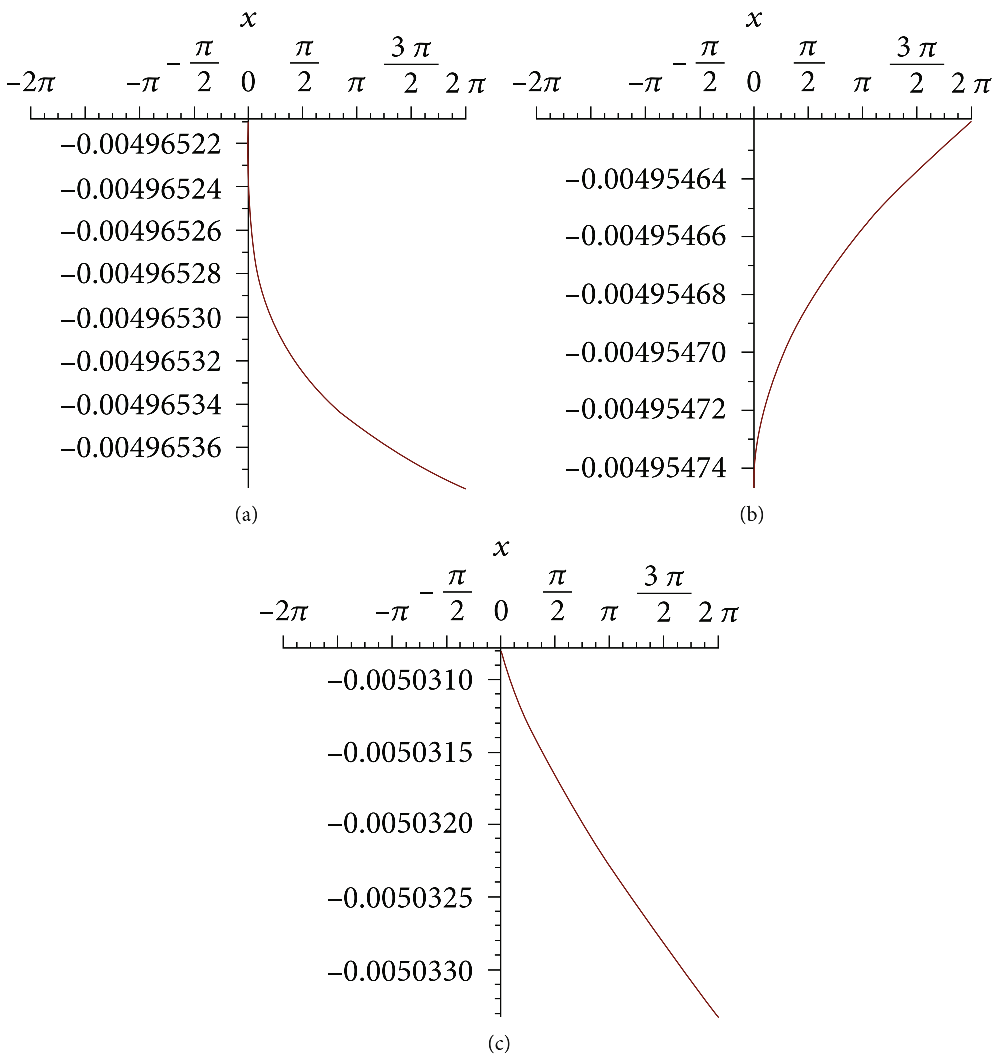

Figure 2: The two-dimensional images of $u_{11}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = \pi/4$, $\sigma = \pi/4$, $s = 0$, $C_2 = 0$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. The above three graphs show corresponding wave propagation.

Case 3. If $\sigma = 0$ and $\rho \neq 0$, $\mu = 0$, under this assumption, solving the above system equation (33), we obtain

$$\begin{cases} \alpha_0 = -\dfrac{c^2 + k^2 + r}{ck}, \\ \alpha_1 = 0, \\ \alpha_2 = -12\rho^2 ck, \\ \beta_1 = 0, \\ \beta_2 = 0, \end{cases} \tag{42}$$

here

$$\begin{gathered} \left(c^2 + k^2 + r\right)^2 = 2cks, \\ \left(\frac{G'}{G^2}\right)(\xi) = -\frac{C_1}{\rho(C_1\xi + C_2)}. \end{gathered} \tag{43}$$

Now, we give the forms of $u(\xi)$:

$$u_{13}(\xi) = \alpha_0 + \alpha_2\left(\frac{G'}{G^2}\right)^2 = -\frac{c^2 + k^2 + r}{ck} - \frac{12ckC_1^2}{(C_1\xi + C_2)^2}. \tag{44}$$

Case 4. If $\mu \neq 0$ and $\Delta = \mu^2 - 4\sigma\rho \geq 0$, now we will divide into two subcases.

Subcase 4.1. If $\sigma = 0$ and $\rho = 0$, and

$$\left(\frac{G'}{G^2}\right)(\xi) = -\frac{\mu}{2\rho} - \left[\frac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right]. \tag{45}$$

This contradicts the truth that the denominator cannot be zero. So this case cannot happen.

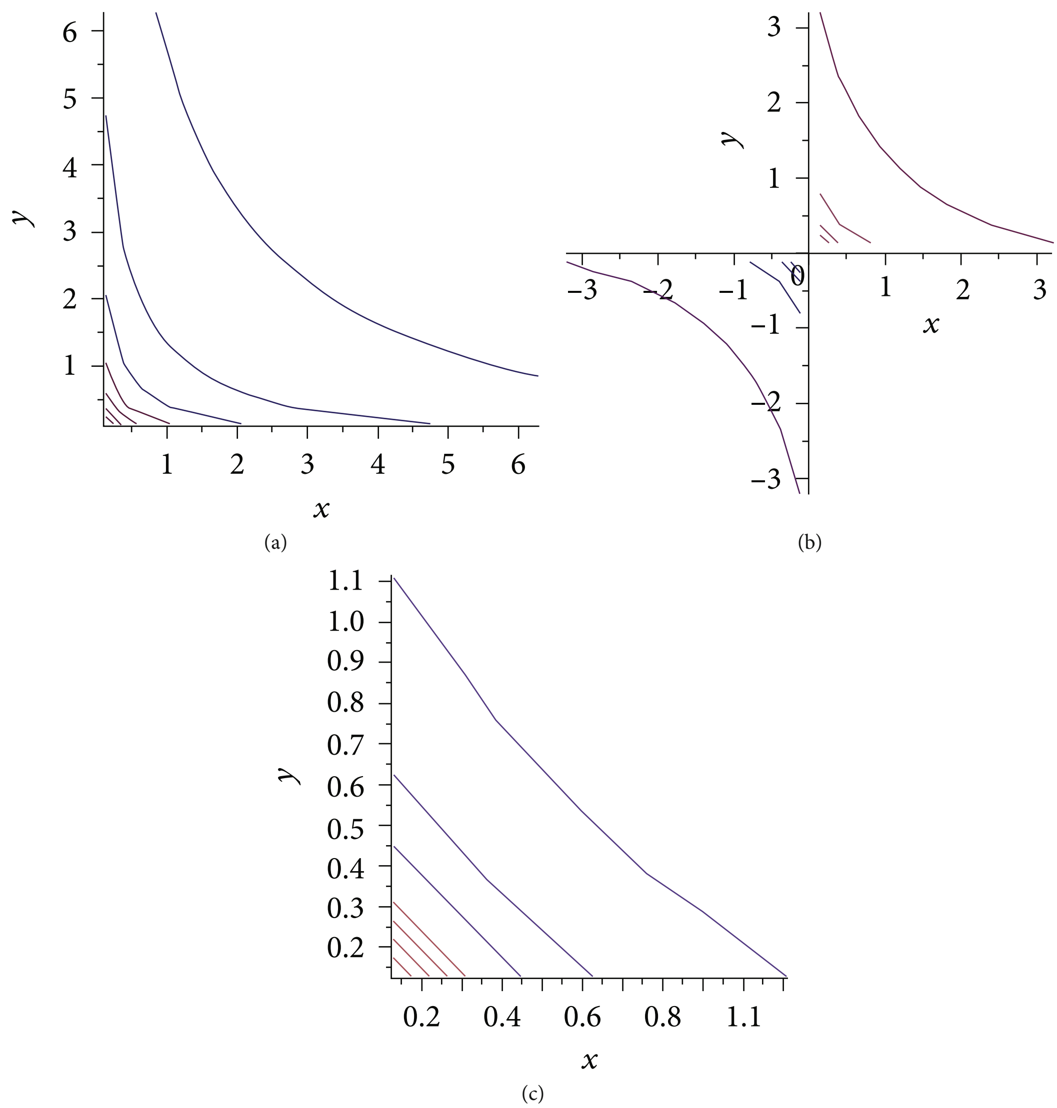

FIGURE 3: The corresponding contour images of $u_{11}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = \pi/4$, $\sigma = \pi/4$, $s = 0$, $C_2 = 0$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$.

If $\sigma = 0$ and $\rho \neq 0$, under this assumption, solving the above system equation (33), we get

$$\begin{cases} \alpha_0 = -u^2 ck - \dfrac{c^2 + k^2 + r}{ck}, \\ \alpha_1 = -12\rho\mu ck, \\ \alpha_2 = -12\rho^2 ck, \\ \beta_1 = 0, \\ \beta_2 = 0, \end{cases} \tag{46}$$

here

$$\left(c^2 + k^2 + r\right)^2 - 2cks = u^4 c^2 k^2,$$
$$\left(\frac{G'}{G^2}\right)(\xi) = -\frac{\mu}{2\rho} - \left[\frac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right]. \tag{47}$$

Now, we give the forms of $u(\xi)$:

$$\begin{aligned} u_{14}(\xi) &= \alpha_0 + \alpha_1\left(\frac{G'}{G^2}\right) + \alpha_2\left(\frac{G'}{G^2}\right)^2 = -u^2 ck - \frac{c^2 + k^2 + r}{ck} \\ &\quad - 12\rho\mu ck\left[-\frac{\mu}{2\rho} - \left(\frac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right] \\ &\quad - 12\rho^2 ck\left[-\frac{\mu}{2\rho} - \left(\frac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right)\right]^2 \\ &= 2\mu^2 ck - \frac{c^2 + k^2 + r}{ck} - 3\mu^2 ck\left(\frac{C_1 \cosh\left(\pm\mu/2\right)\xi + C_2 \sinh\left(\pm\mu/2\right)\xi}{C_2 \cosh\left(\pm\mu/2\right)\xi + C_1 \sinh\left(\pm\mu/2\right)\xi}\right)^2. \end{aligned} \tag{48}$$

Subcase 4.2. If $\sigma \neq 0$, during the system equations, by (1.5), we deduce $\alpha_2 = -12\rho^2 ck$; by (1.6), we deduce $\beta_2 = -12\sigma^2 ck$; and then from (1.4) and (1.7), we get $\alpha_1 = -12\rho\mu ck$, $\beta_1 = -$

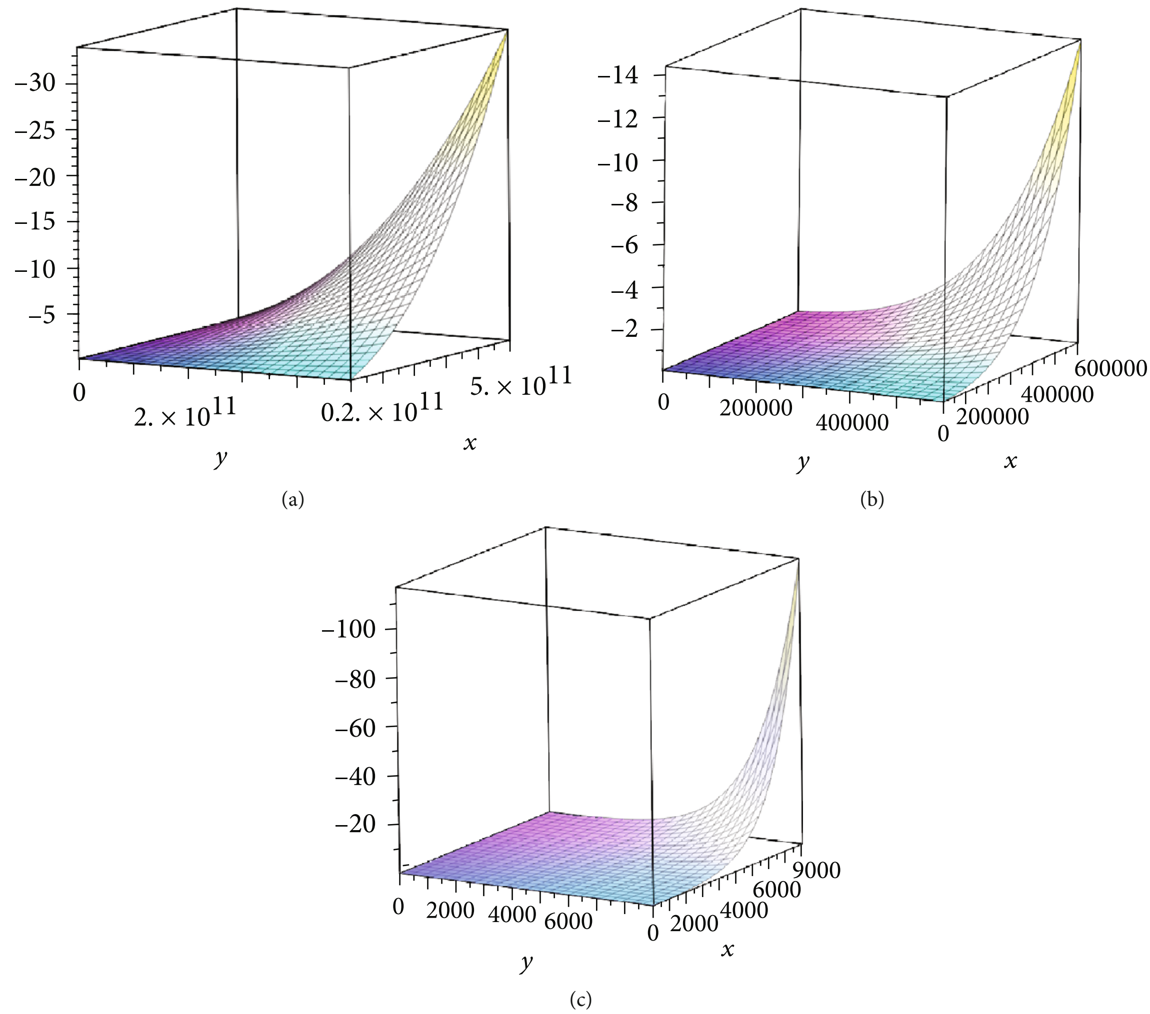

Figure 4: The three-dimensional images of $u_{12}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = -2$, $s = 0$, $C_1 = 0$ take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. The images show annihilation of solitary wave profiles for $u_{12}(\xi)$ accompanied with different α.

$12\mu\sigma ck$, and then from (1.8), we deduce

$$\alpha_0 = -\mu^2 ck + 8\rho\sigma ck - \frac{c^2 + k^2 + r}{ck}, \tag{49}$$

and then from (1.9), we deduce

$$\alpha_0 = -\mu^2 ck + 4\rho\sigma ck - \frac{c^2 + k^2 + r}{ck}, \tag{50}$$

so ρ must be equal to 0. Finally

$$\begin{cases} \alpha_0 = -u^2 ck - \dfrac{c^2 + k^2 + r}{ck} \\ \alpha_1 = 0 \\ \alpha_2 = 0 \\ \beta_1 = -12\mu\sigma ck \\ \beta_2 = -12\sigma^2 ck, \end{cases} \tag{51}$$

here

$$\left(c^2 + k^2 + r\right)^2 - 2cks = u^4 c^4 k^4.$$

$$\left(\frac{G'}{G^2}\right)(\xi) = -\frac{\mu}{2\rho} - \left[\frac{\sqrt{\Delta}\left(C_1 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_2 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cosh\left(\sqrt{\Delta}/2\right)\xi + C_1 \sinh\left(\sqrt{\Delta}/2\right)\xi\right)}\right]. \tag{52}$$

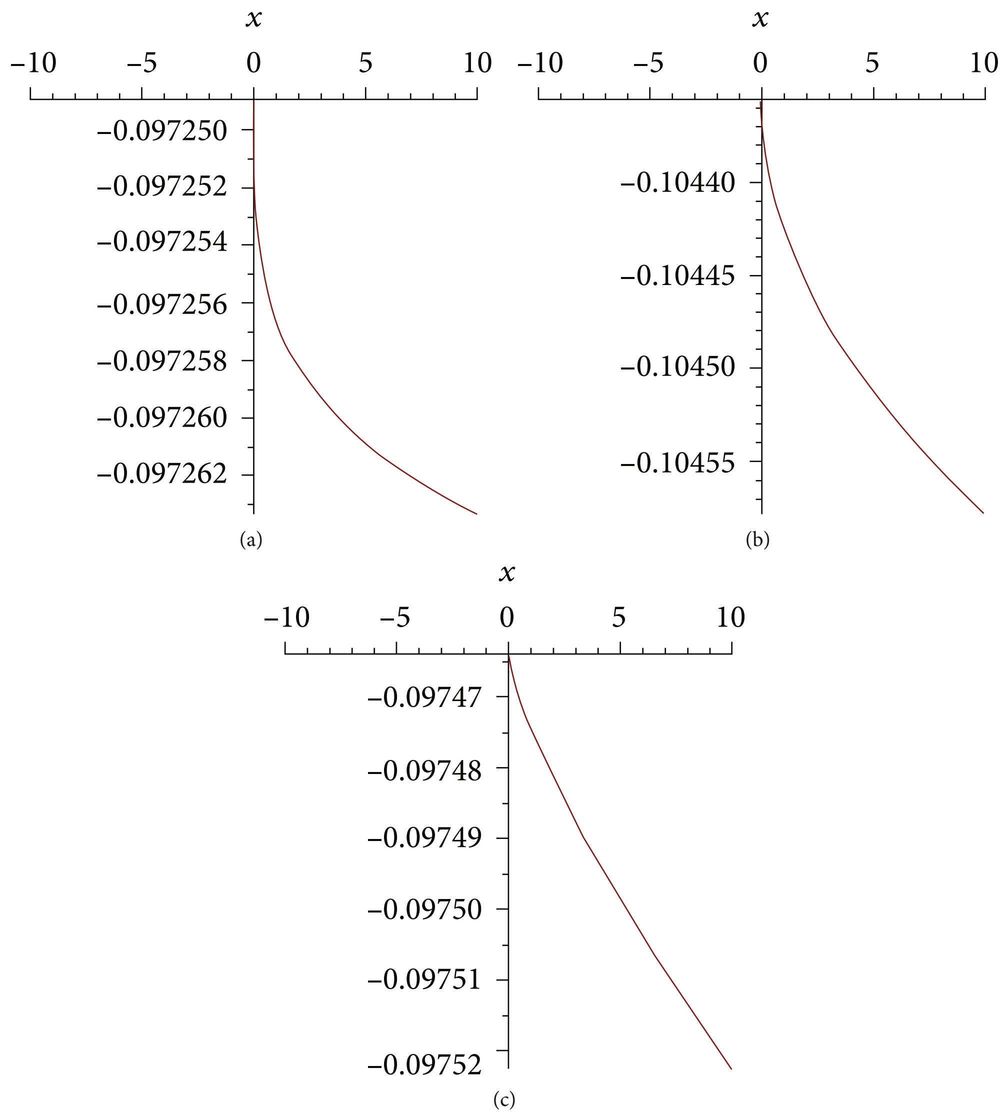

FIGURE 5: The two-dimensional images of $u_{12}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = -2$, $s = 0$, $C_1 = 0$ take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. The three images reveal the wave propagation along x-axis accompanied with different α.

This contradicts the truth that the denominator cannot be zero. So this case cannot happen.

Case 5. If $\mu \neq 0$ and $\Delta = \mu^2 - 4\sigma\rho < 0$, and then the result is the same as Case 4.

Only the case that $\sigma = 0$ and $\rho \neq 0$, under this assumption, solving the above system equation (33), we get

$$\begin{cases} \alpha_0 = -u^2 ck - \dfrac{c^2 + k^2 + r}{ck} \\ \alpha_1 = -12\rho\mu ck \\ \alpha_2 = -12\rho^2 ck \\ \beta_1 = 0 \\ \beta_2 = 0, \end{cases} \tag{53}$$

here

$$\left(c^2 + k^2 + r\right)^2 - 2cks = u^4 c^2 k^2,$$
$$\left(\frac{G'}{G^2}\right)(\xi) = -\frac{\mu}{2\rho} - \left[\frac{\sqrt{-\Delta}\left(C_1 \cos\left(\sqrt{-\Delta}/2\right)\xi - C_2 \sin\left(\sqrt{-\Delta}/2\right)\xi\right)}{2\rho\left(C_2 \cos\left(\sqrt{-\Delta}/2\right)\xi + C_1 \sin\left(\sqrt{-\Delta}/2\right)\xi\right)}\right]. \tag{54}$$

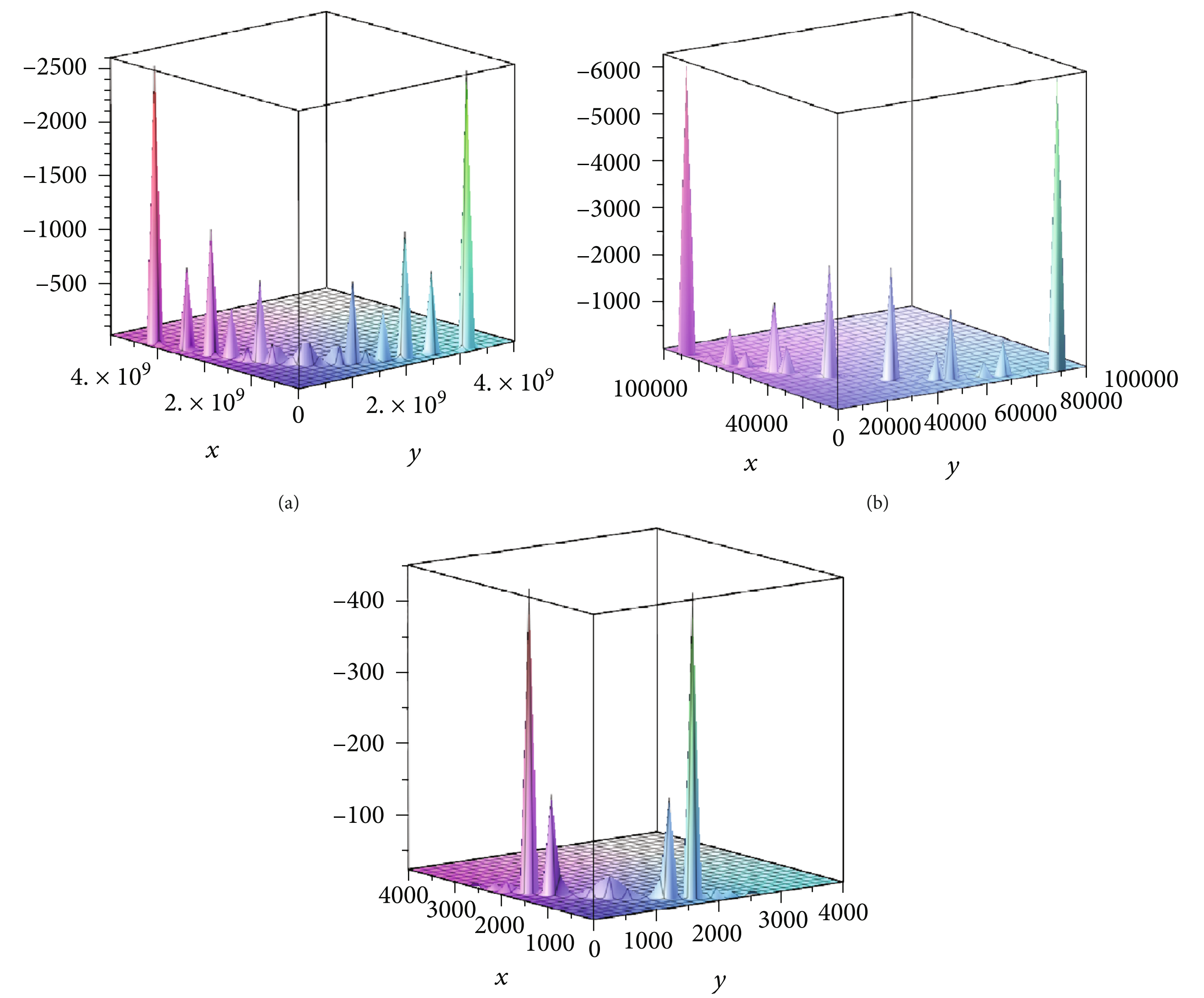

Figure 6: The three-dimensional images of $u_{14}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = 0$, $s = 0$, $\mu = 5$, $C_1 = 3$, $C_2 = 4$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$.

Now, we give the forms of $u(\xi)$:

$$
\begin{aligned}
u_{15}(\xi) = \alpha_0 + \alpha_1\left(\frac{G'}{G^2}\right) + \alpha_2\left(\frac{G'}{G^2}\right)^2 &= -u^2 ck - \frac{c^2+k^2+r}{ck} - 12\rho\mu ck\left[-\frac{\mu}{2\rho} - \frac{\sqrt{-\Delta}\left(C_1\cos\left(\sqrt{-\Delta}/2\right)\xi - C_2\sin\left(\sqrt{-\Delta}/2\right)\xi\right)}{2\rho\left(C_2\cos\left(\sqrt{-\Delta}/2\right)\xi + C_1\sin\left(\sqrt{-\Delta}/2\right)\xi\right)}\right] \\
&- 12\rho^2 ck\left[-\frac{\mu}{2\rho} - \left(\frac{\sqrt{-\Delta}\left(C_1\cos\left(\sqrt{-\Delta}/2\right)\xi - C_2\sin\left(\sqrt{-\Delta}/2\right)\xi\right)}{2\rho\left(C_2\cos\left(\sqrt{-\Delta}/2\right)\xi + C_1\sin\left(\sqrt{-\Delta}/2\right)\xi\right)}\right)\right]^2 = 2\mu^2 ck - \frac{c^2+k^2+r}{ck} \\
&+ 3\mu^2 ck\left(\frac{C_1\cos(\pm i\mu/2)\xi + C_2\sin(\pm i\mu/2)\xi}{C_2\cos(\pm i\mu/2)\xi + C_1\sin(\pm i\mu/2)\xi}\right)^2.
\end{aligned}
\tag{55}
$$

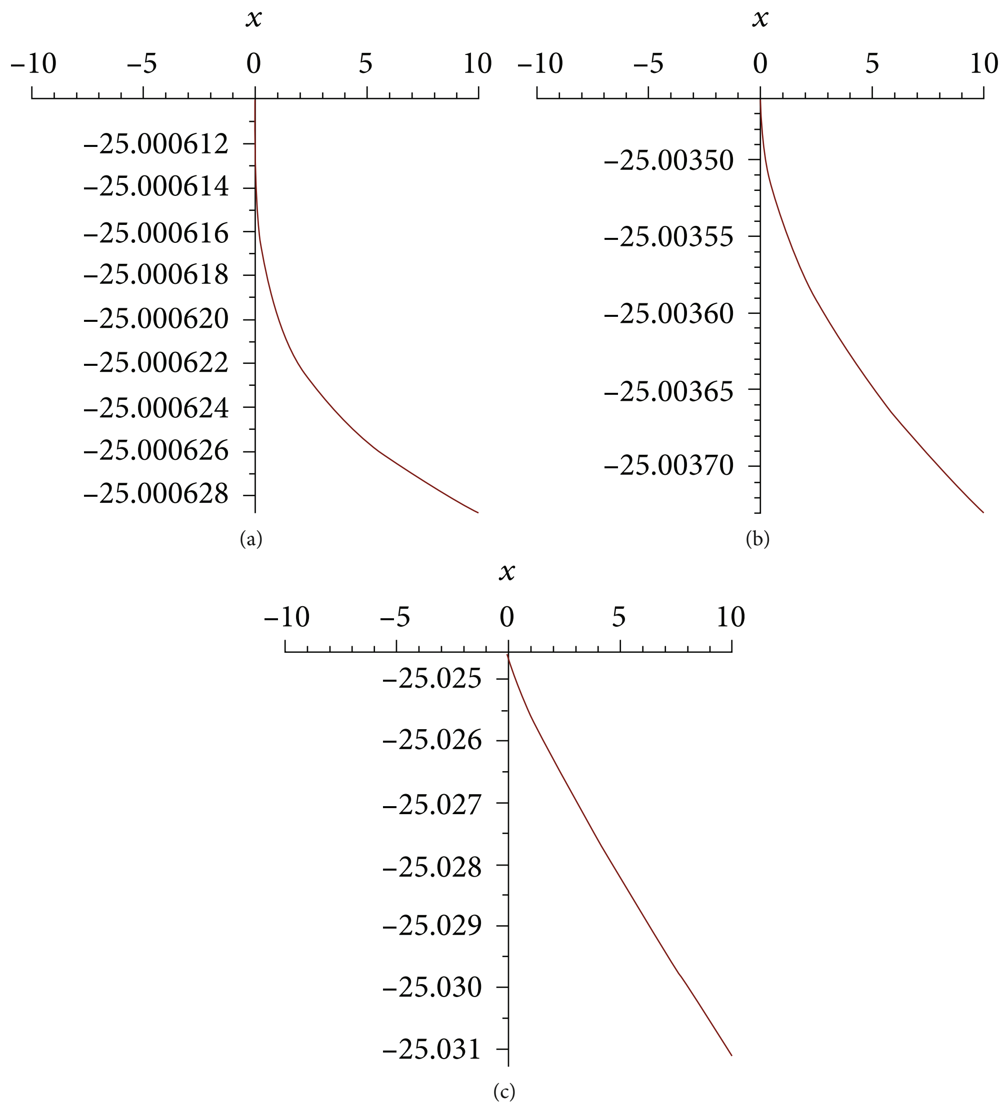

Figure 7: The two-dimensional images of $u_{14}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = 0$, $s = 0$, $\mu = 5$, $C_1 = 3$, $C_2 = 4$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. The three graphs show the curved shape solitary profiles with different α of $u_{14}(\xi)$.

All in the above cases, $\xi = (kx^{\alpha}/\Gamma(1+\alpha)) + (ct^{\alpha}/\Gamma(1+\alpha))$ and C_1 and C_2 are arbitrary constants.

The proof is complete.

5. Proof of Theorem 3

Noting that putting (21) into (5), we have $q = 2$, $p = 1$, $c_{-2} = -12ck$, $c_{-1} = 0$, $c_0 = -c^2 + k^2 + r/ck$, $c_1 = 0$, $c_2 = sck - 2c^2k^2 - (c^2 + k^2 + r)^2/20c^3k^3$, $c_3 = 0$, where c_4 is arbitrary.

Hence, (5) satisfies week $\langle 1, 2\rangle$ condition and we will then give the form of all the meromorphic solutions for (5).

By (14), it is easy to deduce the rational solutions of equation (6) with pole at $\xi = 0$ that

$$u_a(\xi) = \frac{c_{11}}{\xi^2} + \frac{c_{12}}{\xi} + c. \tag{56}$$

Substituting $u_a(\xi)$ into (5), we obtain the following

$$u_{a1}(\xi) = -\frac{12ck}{\xi^2} - \frac{c^2 + k^2 + r}{ck}, \tag{57}$$

where $(c^2 + k^2 + r)^2 = 2cks$. The rational solutions of (5) are as follows:

$$u_{21}(\xi) = -\frac{12ck}{(\xi - \xi_0)^2} - \frac{c^2 + k^2 + r}{ck}, \tag{58}$$

where $(c^2 + k^2 + r)^2 = 2cks$, $\xi_0 \in \mathbb{C}$.

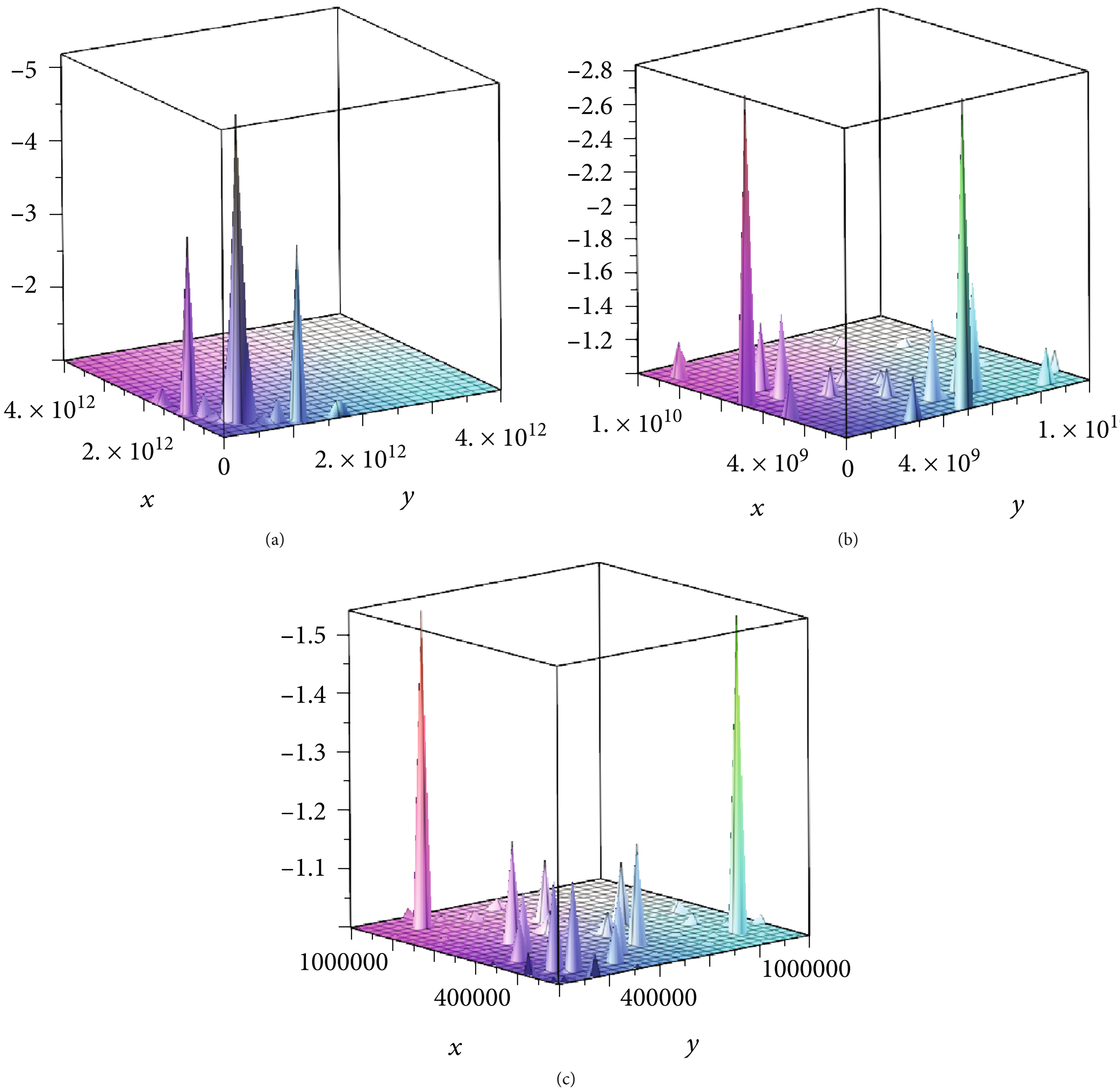

Figure 8: The three-dimensional images of $u_{15}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = 0$, $s = 0$, $\mu = i$, $C_1 = 3$, $C_2 = 4$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. Three graphs demonstrate the multisoliton profiles of $u_{15}(\xi)$ on the domain.

To search for simple periodic solutions, setting $\zeta = \exp(\vartheta\xi)$, $\vartheta \in \mathbb{C}$, putting $u = R(\zeta)$ into (5), then

$$c^2k^2\vartheta^2\left[\zeta R' + \zeta^2 R''\right] + \left(c^2 + k^2 + r\right)R + \frac{1}{2}ckR^2 + s = 0, \quad (59)$$

and putting

$$u_b(\zeta) = \frac{c_2}{(\zeta - 1)^2} + \frac{c_1}{\zeta - \zeta_1} + c_{10} \quad (60)$$

into (55), we get that

$$u_{b1}(\zeta) = \frac{-12ck\vartheta^2}{(\zeta - 1)^2} + \frac{-12ck\vartheta^2}{\zeta - 1} - ck\vartheta^2 - \frac{c^2 + k^2 + r}{ck}, \quad (61)$$

here $\left(c^2 + k^2 + r\right)^2 = 2cks + c^4k^4\vartheta^4$. Substituting $\zeta = e^{\vartheta\xi}$, $\vartheta \in \mathbb{C}$ into the above form, we get all simply periodic solutions of (5) in pole at $\xi = 0$ as follows:

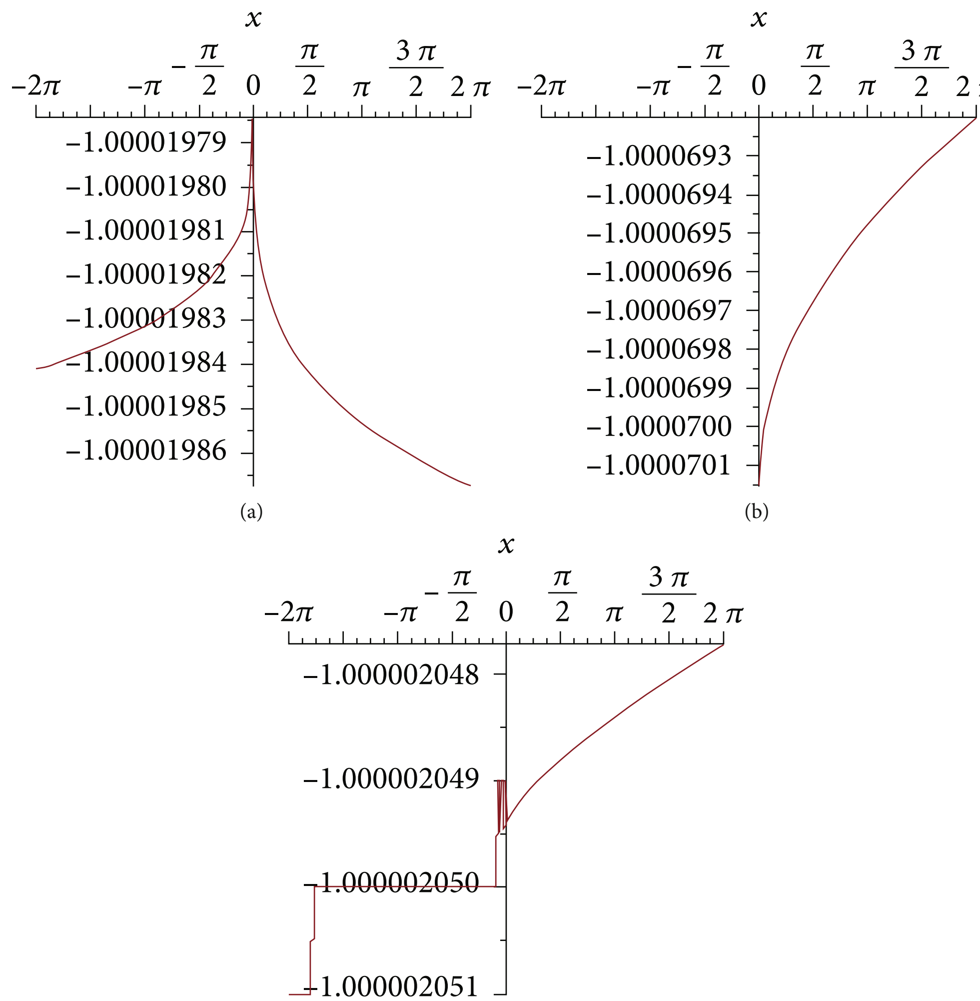

FIGURE 9: The two-dimensional images of $u_{15}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\rho = 2$, $\sigma = 0$, $s = 0$, $\mu = i$, $C_1 = 3$, $C_2 = 4$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. Three graphs show corresponding wave propagation profiles of $u_{15}(\xi)$ on the domain.

$$
\begin{aligned}
u_{b2}(\xi) &= \frac{-12ck\vartheta^2}{\left(e^{\vartheta\xi}-1\right)^2} - \frac{12ck\vartheta^2}{e^{\vartheta\xi}-1} - ck\vartheta^2 - \frac{c^2+k^2+r}{ck} \\
&= \frac{-12ck\vartheta^2\left(e^{\vartheta\xi}\right)}{\left(e^{\vartheta\xi}-1\right)^2} - ck\vartheta^2 - \frac{c^2+k^2+r}{ck} \\
&= -12ck\vartheta^2\coth^2\frac{\vartheta}{2}\xi - ck\vartheta^2 - \frac{c^2+k^2+r}{ck},
\end{aligned}
\tag{62}
$$

So all simply periodic solutions of (5) are obtained by

$$
u_{22}(\xi) = -12ck\alpha^2\coth^2\frac{\vartheta}{2}(\xi-\xi_0) - ck\vartheta^2 - \frac{c^2+k^2+r}{ck}, \tag{63}
$$

where $(c^2+k^2+r)^2 = 2cks + c^4k^4\vartheta^4$, $\xi_0 \in \mathbb{C}$.

From the above part, we are about to obtain the form of the elliptic solution to be determined of (5) with pole at $\xi = 0$ as follows:

$$
u_c(\xi) = c_{-2}\wp(\xi) + c_0. \tag{64}
$$

Substituting $u_c(\xi)$ into the (5), then we obtain that

$$
u_{c1}(\xi) = -12ck\wp(\xi) - \frac{c^2+k^2+r}{ck}, \tag{65}
$$

here $2cks = -12c^4k^4g_2 + (c^2+k^2+r)^2$.

Therefore, all elliptic solutions of (5) are as follows:

$$
u_{c2}(\xi) = -12ck\wp(\xi-\xi_0) - \frac{c^2+k^2+r}{ck}, \tag{66}
$$

here $\xi_0 \in \mathbb{C}$. We rewrite it to the form

$$
u_{23}(\xi) = -12ck\left\{-\wp(\xi) + \frac{1}{4}\left[\frac{\wp'(\xi)+F}{\wp(\xi)-E}\right]^2\right\} + 12ckE - \frac{c^2+k^2+r}{ck}, \tag{67}
$$

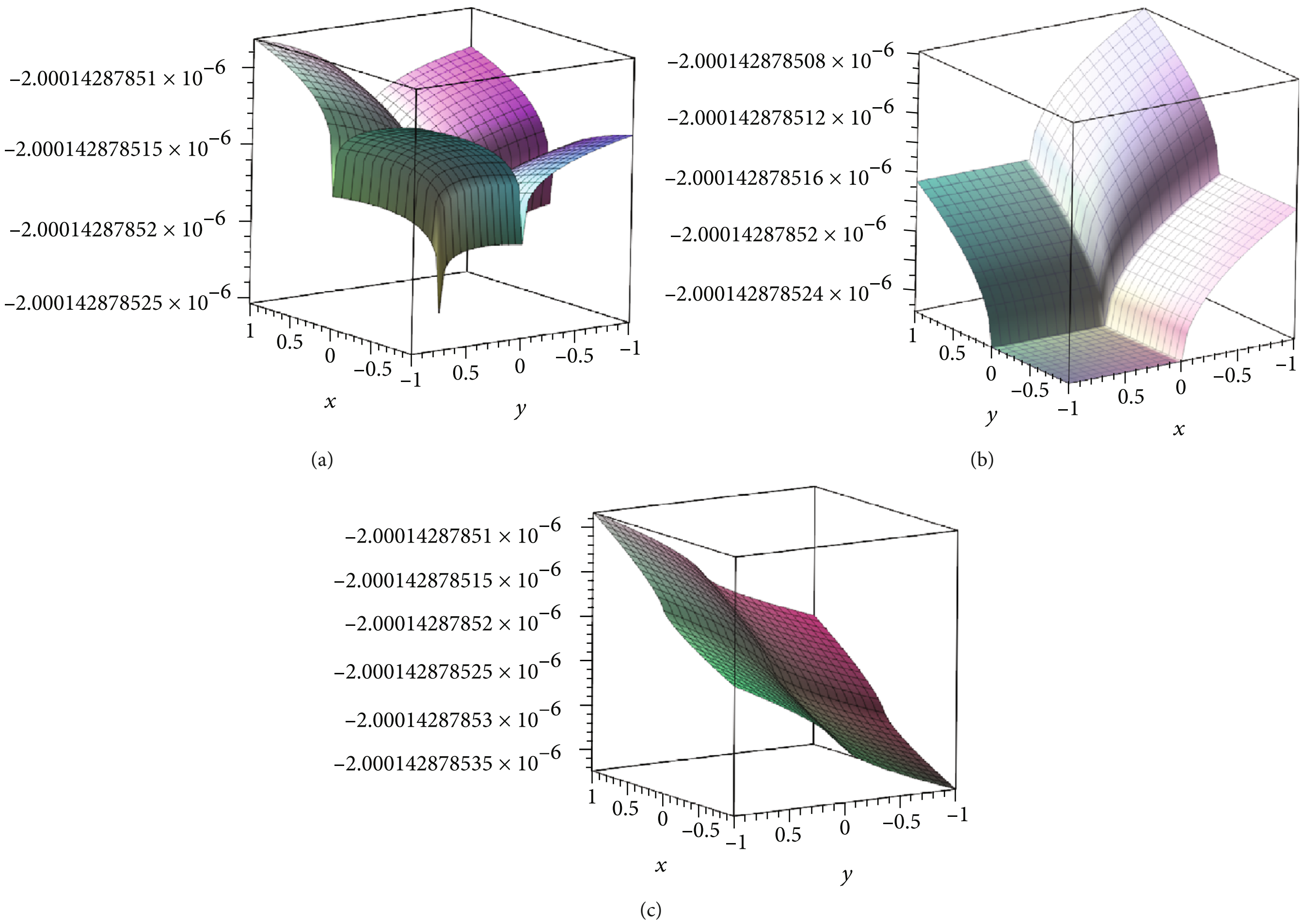

FIGURE 10: The three-dimensional images of $u_{22}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\xi_0 = 1 - 20\sqrt{7}$, $\vartheta = 1/\sqrt{7}$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. Three graphs demonstrate the annihilation of soliton profiles of $u_{22}(\xi)$ on the domain.

here $2cks = -12c^4k^4g_2 + (c^2 + k^2 + r)^2$, $F^2 = 4E^3 - g_2E - g_3$, g_3 and E are arbitrary.

The proof is finished.

6. Accuracy Analysis of Error Function (AAEF)

From eq. (8), it is easy to see the smallness of the AAEF indicating that the approximate solution and exact solution are very close. Thus, AAEF can be used to make an estimate of the accuracy of the approximate solution.

Now, we take some specific parameters in some solutions of eq. (6) and use the Maple software to reckon the AAEF as follows:

For the part solutions in eq. (6), some values of the AAEF are given in Tables 1, 2, and 3, respectively. It is obvious to find that the maximum values of the AAEF are 10^{-9}. It means that the obtained solutions are very near the exact solution.

Obviously, selecting the parameters is important in the calculation of AAEF. From the three tables, AAEF for the solution$u_{22}(\xi)$is the closest to the exact solution, because the value of the AAEF is10^{-18}.

If we take another parameter, it is possible to obtain exact solutions close to the other parameters.

7. Comparison

In this work, with the aid of Maple software, using the extended complex method, we found double periods and single period function solutions of SRLW equation. We also can find more solutions by the (G'/G^2)-expansion method. In 2021, M. A. Khan et al. [32] applied the new auxiliary method to solve for the SRLW. M. A. Khan et al. [32] found the trigonometric and hyperbolic function solutions for the SRLW. However, we can obtain double periods Weierstrass elliptic function solutions just by complex method. Within our cognitive range, the double periods Weierstrass elliptic function solutions got for SRLW in this article mostly have not been reported in the literature. From the level of computational complexity, the extended complex method is more concise and clear. Although (G'/G^2)-expansion method is relatively complex in computation, more formal solutions are obtained.

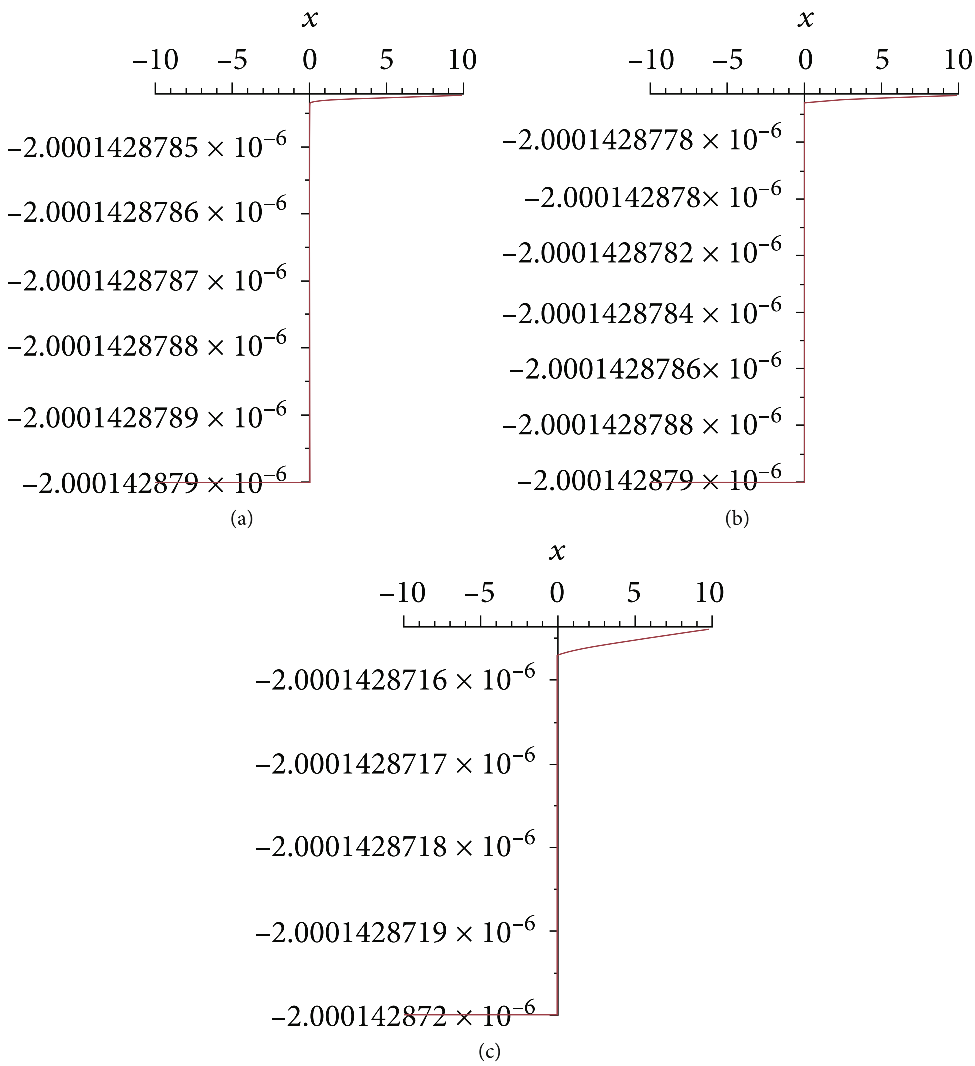

Figure 11: The two-dimensional images of $u_{22}(\xi)$ by considering the values $c = 0.001$, $k = 0.001$, $\xi_0 = 1 - 20\sqrt{7}$, $\vartheta = 1/\sqrt{7}$, $y = 10000$; from (a) to (c), α take the following three different values: $\alpha = 0.25$, $\alpha = 0.5$, $\alpha = 0.75$. Three graphs show corresponding wave propagation profiles of $u_{22}(\xi)$ on the domain.

In 2021, S. C. Ünal et al. [34] got the exact solutions of SRLW equation inspired by a direct method based on the ideas of Jacobi elliptic functions; furthermore, they also get some general form solutions which include rational, single periodic trigonometric and hyperbolic, and double periodics Jacobi elliptic functions. It is well known that the connection between the Jacobian elliptic functions $cn(\xi, m)$ and Weierstrass elliptic function solutions $\wp(\xi)$ is

$$\wp(\xi, g_2, g_3) = e_2 - (e_2 - e_3)cn^2(\sqrt{e_1 - e_3}\,; m), \qquad (68)$$

here $m^2 = (e_2 - e_3)/(e_1 - e_3)$ is the modulus number of Jacobian elliptic functions, and $e_i (i = 1, 2, 3, e_1 \geq e_2 \geq e_3)$ are the roots of equation $4\xi^2 - g_2\xi - g_3 = 0$. It is well known that if $m \longrightarrow 1$ and $e_2 \longrightarrow e_1$, then $cn(\xi\,; m) \longrightarrow \text{sech}\,(\xi)$, or if $m \longrightarrow 0$, then $cn(\xi\,; m) \longrightarrow \cos\,(\xi)$. If $0 < m < 1$, then Jacobian elliptic function is the double periodic meromorphic function and cannot degenerate into single periodic trigonometric and hyperbolic meromorphic functions. Equation (68) is the important bridge linking among the Weierstrass elliptic function, hyperbolic function, trigonometric function, and Jacobian elliptic functions.

Although the means, methods, and ideas of the above methods for studying the exact solutions of SRLW equation vary, the final results obtained still have important relevance and also play important implications for indicating the deep mechanisms of physical phenomena and giving tedious solutions to FNPDE. The extended method enriches the study of the described equations for FNPDE.

8. Computer Simulations

In this section, we are trying to explain the results obtained according to two different methods through computer simulation images, and further analyze the nature of the simple periodic solutions (Figures 1–3) $u_{11}(\xi)$, (Figures 4 and 5)

$u_{12}(\xi)$, (Figures 6 and 7) $u_{14}(\xi)$, (Figures 8 and 9) $u_{15}(\xi)$, and (Figures 10 and 11) $u_{22}(\xi)$ in the SRLW equation.

The three graphs show the cuspon's shape with different α of $u_{14}(\xi)$.

9. Conclusions and Future Outlook of the Study

The extended complex method and the (G'/G^2)-expansion method are very effective tools for seeking the exact solutions of FNPDE. By traveling wave transformation, many of the FNPDE can be converted into IOODE similar to equation (9). In this paper, we employed the extended complex method and the (G'/G^2)-expansion method to seek the exact solutions about SRLW equation. By traveling wave transformation, we can reduce the dimensions of the FNPDE to IOODE related to mathematical physics and engineering. The results of the full text eloquently prove the above methods are very efficient and powerful in solving the exact solutions of SRLW equation now and in the future. We can apply these ideas and methods of this research to other FNPDE.

It is well known that [46] $\wp(z) := \wp(z, g_2, g_3)$ have one successive degeneracy to simple periodic functions according to

$$\wp\left(\xi, 3d^2 - d^3\right) = 2d - \frac{3dd}{2}\coth^2\sqrt{\frac{3d}{2}}\xi, \qquad (69)$$

if one root e_j is double $(\Delta(g_2, g_3) = 0)$. Also $\wp(z) := \wp(z, g_2, g_3)$ have another one successive degeneracy to rational function ξ according to

$$\wp(\xi, 0, 0) = \frac{1}{z^2}, \qquad (70)$$

if one root e_j is triple $(g_2 = g_3 = 0)$. It shows that there exist simple periodic solutions $u_{22}(\xi)$ which is not degenerated successively to the double periods elliptic function.

We are confident that the extended complex methods and (G'/G^2)-extension methods still have broad and bright applications in the future. Both of these methods are effective tools for obtaining FNPDE exact solutions.

In the future, we may consider the simplified improved tan $((\phi(\xi))/2)$-expansion method (SITEM) [47] and generalized direct algebraic method [48] to further investigate the SRLW equation. In 2021, Md. Rezwan Ahamed Fahim et al. [49] used the sine-Gordon-expansion approach for finding the exact solutions of the traveling waves about (3+1)-dimensional Kadomtsev-Petviashvili and the modified KdV-Zakharov-Kuznetsov equations. In 2021, W. X. Ma et al. analyzed soliton solutions and verified the Hirota N-soliton condition for the B-type Kadomtsev-Petviashvili equation, the (2+1)-dimensional KdV equation, the Kadomtsev-Petviashvili equation, and the (2+1)-dimensional Hirota-Satsuma-Ito equation [24, 26], within the Hirota bilinear formulation. In 2021, M. Hafiz Uddin et al. [50] employ the double $(G'/G, 1/G)$-expansion method to investigate the new exact solutions to some fractional nonlinear evolution solutions. This is also another research direction that we will focus on and have great interest in the future.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

The work presented in this paper is supported by NSFC (11326083), Biomedical science and technology support special projects: Intelligent Lower Extremity Rehabilitation Robot (19441914900) and Leading Academic Applied Mathematical of Shanghai Dianji University (16JCXK02). We thank the anonymous reviewers for their positive advices and valuable comments.

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