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Statistical Analysis of Hourly Solar Radiation in Kumasi –Ghana: Bayesian Approach

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

The solar radiation has been known to have skewed distribution rather than normally distributed irrespective of how large the sample size. The Bayesian statistical analysis of the solar radiation sort to find out from the Bayesian perspective how solar radiation is distributed in Ghana. The paper assumed a beta distribution as the conjugate prior for the solar radiation. The posterior distribution for p where p is the probability of sunshine was determined using the Bernoulli probability distribution. In this paper the random variable X represents the event of having a high or low sunshine base on the threshold of $120kWhm^{-2}$. A randomly selected sample of size of 1500 from each month of the year was used in the analysis. Based on the threshold value the total number of sunshine hours was calculated to help in the computation of posterior beta distribution parameters. The Bayesian analysis from the month of January through December were found to converge both for the prior and posterior mean and variances at a tolerance level of 0.0001 and

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0.00001 respectively after twenty iterations. The paper confirms that there are two clusters of which the solar radiation can be placed and that these cluster all converges after twenty iteration. The prior means and posterior mean converges to 0.86000 and .76000 at the tolerance level of 0.0001 respectively while the prior and posterior variances for cluster one and two converged to 6.0E-06 and 4.0E-06 respectively at 0.000001 tolerance level. The papers finally conclude that the clusters have a greater possibility (.86000 and 0.76000) of sunshine. This is also an indication that Kumasi has higher possibility of sunshine.

Keywords: Solar radiation; Bayesian statistical analysis; beta distribution; Bernoulli probability distribution; prior and posterior mean and variances.

1. INTRODUCTION

The prior knowledge of the solar radiation and its abundance in an area is of paramount importance in accessing the potential use in solar energy system Saunier et al. [1]. The solar energy received at the earth surface is subject to seasonal, daily monthly and annual variations. Arthur and Oduro, [2].

Some practical problem of statistical inference may require that decision is taken concerning the parameters of the population parameter instead of finding estimate for them. Arthur et al. [2]. Classical hypothesis has been used to study the significant differences in the various month of the years and has been shown to have significantly different with two major clusters in the year Arthur and Oduro, [2].

The normal probability distribution has been found not to describe the distribution of solar radiation in Ghana and other part of the tropics Akuffo and Brew-Hammond, [3] Yilmaz et al. but various month of the year is described by different probability distribution. Arthur et al. [4].

2. REVIEW OF PROBABILITY THEORY

A Probability Space is a triple (Ω, F, P) consisting of a set Ω called the sample space, a σ-algebra F consisting of subsets of Ω (these subsets are called events) and a measure P (called probability measure) on (Ω, F) such that $\mathrm{P}(\Omega)=1$ called the probability measure.

2.1 Random Variables

Let (Ω, F, P) be a probability space and (Y, G) be a measurable space then the random variable X is defined as a measurable function $X:\Omega \to Y$.

Where $\mathrm{Y} \subseteq \Re$ and G is a σ-algebra of Y consisting of subsets of Y

A random variable X is said to be continuous if there exists a function *f*, called the probability density function or (probability distribution function (pdf)) of X and a function F called the cumulative distribution function (cdf), such that $F(x_2)-F(x_1)=\int_{x_1}^{x_2} f(x)dx = P\{x_1 \le X \le x_2\}$

A random variable X will be called discrete if there exists a finite or countable set U of real numbers with $U=\{x_1, x_2, \ldots x_n\}$ such that $P\{U\}=\sum_{x_i \in U} P(X=x_i)=1$. The probability distribution function (pdf) of a discrete random variable X and the cumulative distribution function (cdf), F are given by $f(x_i)=P(X=x_i)$ and $F(x_n)=\sum_{k=0}^{n} f(x_k)$

If X is a random variable such that $f(x)$ is the probability distribution function then the expectation *E* and the variance *Var* of *X* are given as

$$E(H)=\begin{cases}\sum_{x} hf(h) \text{ if H is discrete}\\ \int_{-\infty}^{\infty} hf(h)\,dx \text{ if H is continious}\end{cases}$$

$$Var(H)=\begin{cases}\sum_{h}\left(h-E(h)\right)^2 f(h) \text{ if H is discrete}\\ \int_{-\infty}^{\infty}\left(h-E(H)\right)^2 f(h)\,dh \text{ if H is continious}\end{cases}$$

2.2 Conditional Probability

Let $H=\{H_1,H_2,....\}$ be a positive partition of the sample space Ω and let A,B be two events with $P(B)>0$, then $P(A|B)=\sum_{j=1}P(A|B\cap H_j)P(H_j|B)$

Proof

Using the expression on the right hand side of the above equation it can be deduced that

$$\sum_{j=1}P(A|B\cap H_j)P(H_j|B)=\sum_{j=1}\frac{P(A\cap B\cap H_j)}{P(B\cap H_j)}\times\frac{P(B\cap H_j)}{P(B)}$$

$$\frac{1}{P(B)}\sum_{j=1}P(A\cap B\cap H_j)=\frac{1}{P(B)}P(A\cap B)\cap\bigcup_{j=1}H_j=\frac{P(A\cap B)}{P(B)}=P(A|B)$$

2.3 Bayes' Formula

We shall now consider a question opposite to the total probability formula. Given that the event occurred, what is the probability of the event in the partition ? The answer is contained in the theorem below.

Theorem (Bayes' formula). *Let* $H=\{H_1,H_2,....\}$ be a positive partition of Ω and A be any event with $P(A)>0$ Then for any event H_k of the partition H we have

$$P(H_k|A)=\frac{P(A|H_k)P(H_k)}{P(A|H_1)P(H_1)+P(A|H_2)P(H_2)+...+P(A|H_n)P(H_n)}$$

3. THE BAYESIAN APPROACH

The Bayesian approach to statistical inference is based upon Bayes' theorem (Bayes 1763), Let consider the problem of finding a point estimate of the parameter θ for the population with distribution $f(x|\ \theta\ )$ given θ. Denote $\pi(\ \theta\ )$ the prior distribution of θ.

Suppose that a random sample of size n, denoted by $X=(x_1,x_2,...,x_n)$ is observed. For continuous distribution the bayes' theorem is defined as follows $\pi(\theta\ |x)\propto f(x\ |\theta)\pi(\theta)$

Definition: The distribution of θ, given data x ,which is called the posterior distribution is given by. $\pi(\theta\ |x)=\frac{f(x|\theta)\pi(\theta)}{g(x)}$ Where $g(x)$ is the marginal distribution of x.

The marginal distribution in the above definition can be calculated using the following formula

$$g(x)=\begin{cases}\sum_{\theta} f\left(x\,|\theta\right)\pi(\theta), & \theta \text{ is discrete}\\ \int_{-\infty}^{\infty} f\left(x\,|\theta\right)\pi(\theta), & \theta \text{ is continous}\end{cases}$$

3.1 Principles of Bayesian Statistical Analysis

The Bayesian statistical analysis is focused on four basic principles.

The specification of an objective probability model of the trials in terms of some unknown parameters. Base on the subjective beliefs about the unknown parameters. These original beliefs are called the prior probabilities. The prior belief is then update in light of the information contained in the sample by applying Bayes rule. The updated beliefs are called the posterior probabilities. Decisions are based on the updated beliefs (i.e., the posterior probabilities).

It is important to remember that, the prior distribution shows your beliefs about different parameter values before seeing any data. The likelihood essentially shows what the observed data tells about how probable different parameter values are. The posterior probability combines the information in these two distributions, and shows your updated beliefs about parameter values after having seen the data. In frequentist statistics, the goal is to obtain good point estimates of parameter values.

In the world of Bayesian statistics, the goal is to obtain a probability distribution over all possible parameter values. This is the posterior probability distribution. It shows how uncertain we are about the parameter value, and can be used as the basis for asking many different questions.

3.2 Bayesian Prior, Posteriors and Estimators

If $\Phi_1,\Phi_2,...,\Phi_N$ denote the random variables associated with a sample of size n .Let the notation $L(\phi_1,\phi_2,...,\phi_n|\theta)$ denote the likelihood of the sample. In the discrete case, this function is defined to be the joint probability $P\left(\Phi_1=\phi_1,\Phi_2=\phi_2,...\Phi_n=\phi_n\right)$ and in the continuous case, it is the joint density of $\Phi_1,\Phi_2,...,\Phi_N$ evaluated at $\phi_1,\phi_2,...,\phi_n$ .The parameter θ is included among the argument of $L(\phi_1,\phi_2,...,\phi_n|\theta)$ to denote this function depends explicitly on the value of some parameter θ.

In Bayesian approach, the unknown parameter θ is viewed to be a random variable with a probability distribution called the prior distribution of θ .This prior distribution of θ is specified before any data are collected and provides a theoretical description of information about θ that was available before any data were obtained Feuillard et al. [5]. In our initial discussion we will assume that the parameter θ has a continuous distribution with density $g\left(\theta\right)$ that has no unknown parameters. Ussher, [6]. Using the likelihood of the data and the prior on θ , it follows that the joint likelihood $\Phi_1,\Phi_2,...\Phi_n,\theta$ is $f\left(\phi_1,\phi_2...\phi_n,\theta\right)=L\left(\phi_1,\phi_2...\phi_n|\theta\right)\times g\left(\theta\right)$ And that the marginal density or mass function of $\Phi_1,\Phi_2,...,\Phi_N$ is

$$m\left(\phi_1,\phi_2...\phi_n,\theta\right)=\int_{-\infty}^{\infty} L\left(\phi_1,\phi_2...\phi_n|\theta\right)\times g\left(\theta\right)d\theta$$

also the posterior density function of $\theta|\phi_1,\phi_2...\phi_n$ is

$$g*\left(\theta|\phi_1,\phi_2...\phi_n\right)=\frac{L\left(\phi_1,\phi_2...\phi_n|\theta\right)\times g\left(\theta\right)}{\int_{-\infty}^{\infty} L\left(\phi_1,\phi_2...\phi_n|\theta\right)\times g\left(\theta\right)d\theta}$$

Let $\Phi_1,\Phi_2,...,\Phi_N$ denote a random sample from a Bernoulli distribution of n observed morning where $P(\Phi_i=1)=p$ and $P(\Phi_i=0)=1-p$ and assume that the prior distribution for p is $beta(\alpha,\beta)$ Robert et al. [7].

4. BETA AS CONJUGATE PRIOR

The paper assumed a beta distribution as the conjugate prior for the solar radiation .The posterior distribution for p where p is the probability of sunshine is to be determined using the Bernoulli probability distribution Hollands and Huget [8]. The Bernoulli probability function can be written as $P(y_i|p)=p^{y_i}(1-p)^{1-y_i}$, $y_i=0,1,$ the likelihood $L\left(y_1,y_2,....,y_n|p\right)$ is

$$
\begin{aligned}
L\left(y_1, y_2, \ldots., y_n \mid p\right) &= p\left(y_1, y_2, \ldots., y_n \mid p\right) \\
&= p^{y_1}(1-p)^{1-y_1} \times p^{y_2}(1-p)^{1-y_2} \times \ldots . \times p^{y_n}(1-p)^{1-y_n} \\
&= p^{\sum y_i}(1-p)^{1-\sum y_i}, \qquad y_i = 0,1 \text{ and } 0 < p < 1
\end{aligned}
$$

The joint probability distribution is

$$
\begin{aligned}
f\left(y_1, y_2, \ldots., y_n, p\right) &= L\left(y_i, y_2, \ldots., y_n \mid p\right) \times g(p) \\
&= p^{\sum y_i}(1-p)^{n-\sum y_i} \times \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1}(1-p)^{\beta-1} \\
&= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}
\end{aligned}
$$

And the marginal density function is

$$
\begin{aligned}
m\left(y_1, y_2, \ldots., y_n\right) &= \int_0^1 \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1} dp \\
&= \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \frac{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)}{\Gamma(n+\alpha+\beta)}
\end{aligned}
$$

The posterior density of p is given by

$$
g^*\left(\theta \mid y_1, y_2 \ldots y_n\right) = \frac{L\left(y_1, y_2 \ldots y_n \mid \theta\right) \times g(\theta)}{\int_{-\infty}^{\infty} L\left(y_1, y_2 \ldots y_n \mid \theta\right) \times g(\theta) d\theta}
$$

Hence,

$$
\begin{aligned}
g^*\left(p \mid y_1, y_2, \ldots, y_n\right) &= \frac{\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}}{\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \times \dfrac{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)}{\Gamma(n+\alpha+\beta)}} \\
&= \frac{\Gamma(n+\alpha+\beta)}{\Gamma\left(\sum y_i+\alpha\right)\Gamma\left(n-\sum y_i+\beta\right)} \times p^{\sum y_i+\alpha-1}(1-p)^{n-\sum y_i+\beta-1}, \qquad 0<p<1
\end{aligned}
$$

The posterior beta density function is with the parameters $\alpha^* = \sum y_i + \alpha$ and $\beta^* = n - \sum y_i + \beta$

5. APPLICATION OF BAYESIAN ANALYSIS ON SOLAR RADIATION

In the Bayesian analysis we use the random variable X which represents the event of having a high or low sunshine.

Thus $X = \begin{cases} 0 \text{ low sunshine} \\ 1 \text{ high sunshine} \end{cases}$

The values of X are generated based on a given threshold below which we describe the sunshine level as being low and as high otherwise. The threshold value used in this analysis is $120kWhm^{-2}$. A random sample of 1500 sunshine hours were selected and based on the above mentioned threshold the monthly average number of high and low sunshine levels was computed.

The beta distribution was used as prior distribution and the Bernoulli probability distribution as a likelihood function Ramakant Khazanie [9]. The posterior probability distribution is derived in section 2.46.

Bayesian analysis for the various months of the year was conducted to estimate the posterior expectation and variance of the beta distribution Olseth and Skartvieit, [10] for number of iterations until these iterations are converging to a certain value given a tolerance level. The analysis started with initial parameter values of alpha and beta to compute the prior means and prior variance .The posterior parameters are then computed based on the sample size and the number of high sunshine hours. The posterior parameters were then used as the prior parameters to compute the mean and variance. The procedure is repeated for twenty iterations for all the months of the year.

5.1 Beta Distribution

A random variable X is said to have beta distribution with parameters $\alpha > 0$ $\beta > 0$ if the density of X is

$$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} \text{ for } 0 \le x \le 1$$

and $f(x) = 0$ outside the interval $[0,1]$.

The mean and the variance of the beta distribution is given below

$$Mean \text{ denoted } E(X) = \mu = \frac{\alpha}{\alpha+\beta} \text{ and}$$

$$\text{variance denoted Var}(\text{X}) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$$

6. BAYESIAN METHODOLOGY

The prior beta distribution is used to together with the likelihood function to obtain the posterior distribution Bowerman et al. [11]. The beta distribution being a conjugate prior always has a posterior beta distribution with parameters

$$\alpha^* = \sum x + \alpha$$
$$\beta^* = n - \sum x + \beta$$

Where n is the sample size and $\sum x$ is the total number of sunshine hours in the sampled data under the threshold of 120 kwh/m^2.The alpha and beta values were computed and used as the initial guess. The posterior parameters were computed using the formula above. The posterior alpha and beta values were used as the prior alpha and beta values to compute the posterior parameters for that iteration.

The posterior mean and variance for the posterior probability distribution was computed as below for the number of iterations.

$$E(X) = \frac{\alpha^*}{\alpha^* + \beta^*}$$
$$Var(X) = \frac{\alpha^*\beta^*}{(\alpha^*+\beta^*)^2(\alpha^*+\beta^*+1)}$$

7. RESULTS, FINDINGS AND DISCUSSION

A sample of size of 1500 was selected from the various month of the year. Based on the threshold value the total number of sunshine hours is calculated to help in the computation of posterior beta distribution parameters.

7.1 Bayesian Results for January to December

The samples selected from the month of January to the month of December were analyzed using Bayesian approach. Twenty iterations were performed and the results are at the appendix. The results in the Table 1 indicates that the prior means and posterior mean converges to .87000 at a tolerance level of 0.0001 while the prior and posterior variances are converging to 4.0E-06 at 0.000001 tolerance level. The analysis shows that the month of January has a greater possibility (.87000) of sunshine.

The results in Table 2 in the appendix indicates that, at a tolerance level of 0.0001 and 0.00001, the prior means and posterior mean as well as the prior and posterior variances for the month February converges to 0.83000 and 6.0E-06 respectively .

The results for March is presented in Table 3 in the appendix, the result shows that, prior means and posterior mean converges to 0.9000 tolerance level of 0.0001 while the prior and posterior variances are converging to 3.0E-06 at 0.000001 tolerance level.

The results in the Table 4 of the appendix for April reveals that, the prior means and posterior mean converges to 0.86000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

The results in the Table 5 of the appendix for May reveals that, the prior means and posterior mean converges to 0.92000 tolerance level of 0.0001 while the prior and posterior variances are converging to 2.0E-06 at 0.000001 tolerance level.

The results in the Table 6 of the appendix for June reveals that, the prior means and posterior mean converges to 0.77000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

The results in the Table 7 of the appendix for July reveals that, the prior means and posterior mean converges to 0.92000 tolerance level of 0.0001 while the prior and posterior variances are converging to 7.0E-06 at 0.000001 tolerance level.

The results in the Table 8 of the appendix for August reveals that, the prior means and posterior mean converges to 0.55000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

The results in the Table 9 of the appendix for September reveals that, the prior means and posterior mean converges to 0.79000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

The results in the Table 10 of the appendix for October reveals that, the prior means and posterior mean converges to 0.72000 tolerance level of 0.0001 while the prior and posterior variances are converging to 2.0E-06 at 0.000001 tolerance level.

The results in the Table 11 of the appendix for November reveals that, the prior means and posterior mean converges to 0.9000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

The results in the Table 12 of the appendix for December reveals that, the prior means and posterior mean converges to 0.87000 tolerance level of 0.0001 while the prior and posterior variances are converging to 6.0E-06 at 0.000001 tolerance level.

7.2 Bayesian Estimation for Months

As indicated in Tables 1 to 12, the Bayesian analysis was conducted for the various months of the year. The summary of the results are given in Tables 1 to 12 in the appendix. The results presented are the means and the variance for twenty iterations for the various months of the year. The results in Table 1 to 12 indicate that the beta distribution which was used as the prior distribution converged to the mean and variance of the posterior distributions shown. The posterior means of various months indicates that, despite the rain fall patterns in Kumasi there is a higher potential sunshine region.

7.3 Bayesian Analysis for Clusters

The sample selected from the clusters was analyzed using Bayesian approach. Twenty iterations were performed and the results are shown on the Table 13 and Table 14 are for cluster one and cluster two respectively. The results in the Table 13 and Table 14 indicates that the prior means and posterior mean converges to 0.86000 and .76000 at the tolerance level of 0.0001 respectively while the prior and posterior variances for cluster one and two converged to 6.0E-06 and 4.0E-06 respectively at 0.000001 tolerance level. The analysis shows that the clusters have a greater possibility (.86000 and 0.76000) of sunshine. This is also an indication that Kumasi is has higher possibility of sunshine.

8. CONCLUSION

Bayesian analysis on high or low (i.e. respectively above or below a threshold of $120kWhm^{-2}$ hourly solar irradiation) for each month with given prior beta distribution converged to posterior beta distribution after 20 iteration with average mean of 0.86. This shows that the on average the solar irradiation patterns in Kumasi tends to be high frequently. Also the prior variance of the various months of the year converged to the posterior tolerance level of 0.000001.

Comparing the sunshine output of the two clusters indicated that cluster one have the highest hourly solar irradiation output than that of cluster two. The Bayesian analysis of the clusters also confirms that the first cluster have a higher possibility (0.8600) of solar irradiation output as compared with the second cluster (0.76000). This results also confirm the earlier result of Arthur and et al that the month of the year can be put into two clusters and that the first cluster which has February has the highest potentiality of sunshine.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX

Table 1. Bayesian analysis for January

ITERATION S	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	27	7	0.794117647	0.00467128	1332	202	0.868318123	7.44897E-05
2	1332	202	0.868318123	7.44897E-05	2637	397	0.869149637	3.74723E-05
3	2637	397	0.869149637	3.74723E-05	3942	592	0.869430966	2.50321E-05
4	3942	592	0.869430966	2.50321E-05	5247	787	0.869572423	1.87931E-05
5	5247	787	0.869572423	1.87931E-05	6552	982	0.869657552	1.50436E-05
6	6552	982	0.869657552	1.50436E-05	7857	1177	0.869714412	1.25414E-05
7	7857	1177	0.869714412	1.25414E-05	9162	1372	0.869755079	1.07528E-05
8	9162	1372	0.869755079	1.07528E-05	10467	1567	0.869785607	9.41077E-06
9	10467	1567	0.869785607	9.41077E-06	11772	1762	0.869809369	8.36653E-06
10	11772	1762	0.869809369	8.36653E-06	13077	1957	0.869828389	7.53089E-06
11	13077	1957	0.869828389	7.53089E-06	14382	2152	0.869843958	6.84702E-06
12	14382	2152	0.869843958	6.84702E-06	15687	2347	0.869856937	6.27701E-06
13	15687	2347	0.869856937	6.27701E-06	16992	2542	0.869867923	5.79461E-06
14	16992	2542	0.869867923	5.79461E-06	18297	2737	0.869877341	5.38107E-06
15	18297	2737	0.869877341	5.38107E-06	19602	2932	0.869885506	5.02262E-06
16	19602	2932	0.869885506	5.02262E-06	20907	3127	0.869892652	4.70894E-06
17	20907	3127	0.869892652	4.70894E-06	22212	3322	0.869898958	4.43214E-06
18	22212	3322	0.869898958	4.43214E-06	23517	3517	0.869904565	4.18608E-06
19	23517	3517	0.869904565	4.18608E-06	24822	3712	0.869909582	3.9659E-06
20	24822	3712	0.869909582	3.9659E-06	26127	3907	0.869914097	3.76772E-06

Table 2. Bayesian analysis for February

Iterations	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	1	6	0.142857143	0.015306122	1046	461	0.694094227	0.000140801
2	1046	461	0.694094227	0.000140801	2091	671	0.757060101	6.65654E-05
3	2091	671	0.757060101	6.65654E-05	3136	881	0.780682101	4.26126E-05
4	3136	881	0.780682101	4.26126E-05	4181	1091	0.793057663	3.11241E-05
5	4181	1091	0.793057663	3.11241E-05	5226	1301	0.800674123	2.44478E-05
6	5226	1301	0.800674123	2.44478E-05	6271	1511	0.805833976	2.01035E-05
7	6271	1511	0.805833976	2.01035E-05	7316	1721	0.809560695	1.70582E-05
8	7316	1721	0.809560695	1.70582E-05	8361	1931	0.812378546	1.48081E-05
9	8361	1931	0.812378546	1.48081E-05	9406	2141	0.814583875	1.30791E-05
10	9406	2141	0.814583875	1.30791E-05	10451	2351	0.816356819	1.17096E-05
11	10451	2351	0.816356819	1.17096E-05	11496	2561	0.817813189	1.05986E-05
12	11496	2561	0.817813189	1.05986E-05	12541	2771	0.819030825	9.67931E-06
13	12541	2771	0.819030825	9.67931E-06	13586	2981	0.820063983	8.90627E-06
14	13586	2981	0.820063983	8.90627E-06	14631	3191	0.820951633	8.24721E-06
15	14631	3191	0.820951633	8.24721E-06	15676	3401	0.821722493	7.67872E-06
16	15676	3401	0.821722493	7.67872E-06	16721	3611	0.82239819	7.18337E-06
17	16721	3611	0.82239819	7.18337E-06	17766	3821	0.822995321	6.74792E-06
18	17766	3821	0.822995321	6.74792E-06	18811	4031	0.823526837	6.36214E-06
19	18811	4031	0.823526837	6.36214E-06	19856	4241	0.824002988	6.01801E-06
20	19856	4241	0.824002988	6.01801E-06	20901	4451	0.824431997	5.70914E-06

Table 3. Bayesian analysis for March

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	13	1	0.928571429	0.004421769	1363	151	0.900264201	5.92664E-05
2	1363	151	0.900264201	5.92664E-05	2713	301	0.900132714	2.98155E-05
3	2713	301	0.900132714	2.98155E-05	4063	451	0.900088613	1.99179E-05
4	4063	451	0.900088613	1.99179E-05	5413	601	0.900066511	1.49537E-05
5	5413	601	0.900066511	1.49537E-05	6763	751	0.900053234	1.19704E-05
6	6763	751	0.900053234	1.19704E-05	8113	901	0.900044375	9.97942E-06
7	8113	901	0.900044375	9.97942E-06	9463	1051	0.900038045	8.55631E-06
8	9463	1051	0.900038045	8.55631E-06	10813	1201	0.900033294	7.48842E-06
9	10813	1201	0.900033294	7.48842E-06	12163	1351	0.900029599	6.65752E-06
10	12163	1351	0.900029599	6.65752E-06	13513	1501	0.900026642	5.99259E-06
11	13513	1501	0.900026642	5.99259E-06	14863	1651	0.900024222	5.44842E-06
12	14863	1651	0.900024222	5.44842E-06	16213	1801	0.900022205	4.99485E-06
13	16213	1801	0.900022205	4.99485E-06	17563	1951	0.900020498	4.611E-06
14	17563	1951	0.900020498	4.611E-06	18913	2101	0.900019035	4.28193E-06
15	18913	2101	0.900019035	4.28193E-06	20263	2251	0.900017767	3.9967E-06
16	20263	2251	0.900017767	3.9967E-06	21613	2401	0.900016657	3.7471E-06
17	21613	2401	0.900016657	3.7471E-06	22963	2551	0.900015678	3.52685E-06
18	22963	2551	0.900015678	3.52685E-06	24313	2701	0.900014807	3.33104E-06
19	24313	2701	0.900014807	3.33104E-06	25663	2851	0.900014028	3.15584E-06
20	25663	2851	0.900014028	3.15584E-06	27013	3001	0.900013327	2.99815E-06

Table 4. Bayesian analysis for April

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	26	16	0.619047619	0.005484364	1316	226	0.853437095	8.10643E-05
2	1316	226	0.853437095	8.10643E-05	2606	436	0.856673241	4.03497E-05
3	2606	436	0.856673241	4.03497E-05	3896	646	0.857771907	2.68543E-05
4	3896	646	0.857771907	2.68543E-05	5186	856	0.858325058	2.0123E-05
5	5186	856	0.858325058	2.0123E-05	6476	1066	0.858658181	1.60897E-05
6	6476	1066	0.858658181	1.60897E-05	7766	1276	0.858880779	1.34031E-05
7	7766	1276	0.858880779	1.34031E-05	9056	1486	0.85904003	1.14854E-05
8	9056	1486	0.85904003	1.14854E-05	10346	1696	0.859159608	1.00477E-05
9	10346	1696	0.859159608	1.00477E-05	11636	1906	0.859252695	8.92989E-06
10	11636	1906	0.859252695	8.92989E-06	12926	2116	0.859327217	8.03589E-06
11	12926	2116	0.859327217	8.03589E-06	14216	2326	0.859388224	7.30461E-06
12	14216	2326	0.859388224	7.30461E-06	15506	2536	0.859439087	6.69531E-06
13	15506	2536	0.859439087	6.69531E-06	16796	2746	0.859482141	6.17984E-06
14	16796	2746	0.859482141	6.17984E-06	18086	2956	0.859519057	5.73806E-06
15	18086	2956	0.859519057	5.73806E-06	19376	3166	0.85955106	5.35523E-06
16	19376	3166	0.85955106	5.35523E-06	20666	3376	0.85957907	5.02029E-06
17	20666	3376	0.85957907	5.02029E-06	21956	3586	0.85960379	4.72478E-06
18	21956	3586	0.85960379	4.72478E-06	23246	3796	0.859625767	4.46213E-06
19	23246	3796	0.859625767	4.46213E-06	24536	4006	0.859645435	4.22714E-06
20	24536	4006	0.859645435	4.22714E-06	25826	4216	0.859663138	4.01566E-06

Table 5. Bayesian analysis for May

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	15	1	0.9375	0.003446691	1400	116	0.92348285	4.65803E-05
2	1400	116	0.92348285	4.65803E-05	2785	231	0.923408488	2.34422E-05
3	2785	231	0.923408488	2.34422E-05	4170	346	0.923383525	1.56623E-05
4	4170	346	0.923383525	1.56623E-05	5555	461	0.923371011	1.17595E-05
5	5555	461	0.923371011	1.17595E-05	6940	576	0.923363491	9.41378E-06
6	6940	576	0.923363491	9.41378E-06	8325	691	0.923358474	7.84824E-06
7	8325	691	0.923358474	7.84824E-06	9710	806	0.923354888	6.72917E-06
8	9710	806	0.923354888	6.72917E-06	11095	921	0.923352197	5.8894E-06
9	11095	921	0.923352197	5.8894E-06	12480	1036	0.923350104	5.23598E-06
10	12480	1036	0.923350104	5.23598E-06	13865	1151	0.923348428	4.71307E-06
11	13865	1151	0.923348428	4.71307E-06	15250	1266	0.923347057	4.28512E-06
12	15250	1266	0.923347057	4.28512E-06	16635	1381	0.923345915	3.92841E-06
13	16635	1381	0.923345915	3.92841E-06	18020	1496	0.923344948	3.62653E-06
14	18020	1496	0.923344948	3.62653E-06	19405	1611	0.923344119	3.36774E-06
15	19405	1611	0.923344119	3.36774E-06	20790	1726	0.9233434	3.14342E-06
16	20790	1726	0.9233434	3.14342E-06	22175	1841	0.923342771	2.94712E-06
17	22175	1841	0.923342771	2.94712E-06	23560	1956	0.923342217	2.77389E-06
18	23560	1956	0.923342217	2.77389E-06	24945	2071	0.923341723	2.6199E-06
19	24945	2071	0.923341723	2.6199E-06	26330	2186	0.923341282	2.4821E-06
20	26330	2186	0.923341282	2.4821E-06	27715	2301	0.923340885	2.35808E-06

Table 6. Bayesian analysis for June

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	21	18	0.538461538	0.006213018	1075	464	0.698505523	0.00013675
2	1075	464	0.698505523	0.00013675	2129	910	0.700559395	6.90052E-05
3	2129	910	0.700559395	6.90052E-05	3183	1356	0.701255783	4.61445E-05
4	3183	1356	0.701255783	4.61445E-05	4237	1802	0.701606226	3.46614E-05
5	4237	1802	0.701606226	3.46614E-05	5291	2248	0.701817217	2.77546E-05
6	5291	2248	0.701817217	2.77546E-05	6345	2694	0.701958181	2.3143E-05
7	6345	2694	0.701958181	2.3143E-05	7399	3140	0.702059019	1.98456E-05
8	7399	3140	0.702059019	1.98456E-05	8453	3586	0.702134729	1.73706E-05
9	10346	1696	0.859159608	1.00477E-05	11400	2142	0.841825432	9.83204E-06
10	11400	2142	0.841825432	9.83204E-06	12454	2588	0.827948411	9.46951E-06
11	12454	2588	0.827948411	9.46951E-06	13508	3034	0.816588079	9.0535E-06
12	13508	3034	0.816588079	9.0535E-06	14562	3480	0.807116728	8.62824E-06
13	14562	3480	0.807116728	8.62824E-06	15616	3926	0.799099376	8.21468E-06
14	15616	3926	0.799099376	8.21468E-06	16670	4372	0.792225074	7.82229E-06
15	16670	4372	0.792225074	7.82229E-06	17724	4818	0.786265637	7.45473E-06
16	17724	4818	0.786265637	7.45473E-06	18778	5264	0.781049829	7.11271E-06
17	18778	5264	0.781049829	7.11271E-06	19832	5710	0.776446637	6.79549E-06
18	19832	5710	0.776446637	6.79549E-06	20886	6156	0.772354116	6.50162E-06
19	20886	6156	0.772354116	6.50162E-06	21940	6602	0.768691753	6.22936E-06
20	21940	6602	0.768691753	6.22936E-06	22994	7048	0.765395114	5.97695E-06

Table 7. Bayesian analysis for July

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	28	14	0.666666667	0.005167959	1082	427	0.717031146	0.000134369
2	1082	427	0.717031146	0.000134369	2136	840	0.717741935	6.80512E-05
3	2136	840	0.717741935	6.80512E-05	3190	1253	0.717983345	4.55633E-05
4	3190	1253	0.717983345	4.55633E-05	4244	1666	0.718104907	3.42464E-05
5	4244	1666	0.718104907	3.42464E-05	5298	2079	0.718178121	2.74327E-05
6	5298	2079	0.718178121	2.74327E-05	6352	2492	0.718227047	2.28804E-05
7	6352	2492	0.718227047	2.28804E-05	7406	2905	0.71826205	1.96239E-05
8	7406	2905	0.71826205	1.96239E-05	8460	3318	0.718288334	1.71789E-05
9	8460	3318	0.718288334	1.71789E-05	9514	3731	0.718308796	1.52757E-05
10	9514	3731	0.718308796	1.52757E-05	10568	4144	0.718325177	1.37521E-05
11	10568	4144	0.718325177	1.37521E-05	11622	4557	0.718338587	1.25048E-05
12	11622	4557	0.718338587	1.25048E-05	12676	4970	0.718349768	1.1465E-05
13	12676	4970	0.718349768	1.1465E-05	13730	5383	0.718359232	1.05849E-05
14	13730	5383	0.718359232	1.05849E-05	14784	5796	0.718367347	9.83022E-06
15	14784	5796	0.718367347	9.83022E-06	15838	6209	0.718374382	9.17601E-06
16	15838	6209	0.718374382	9.17601E-06	16892	6622	0.718380539	8.60344E-06
17	16892	6622	0.718380539	8.60344E-06	17946	7035	0.718385973	8.09813E-06
18	17946	7035	0.718385973	8.09813E-06	19000	7448	0.718390805	7.64889E-06
19	19000	7448	0.718390805	7.64889E-06	20054	7861	0.718395128	7.24687E-06
20	20054	7861	0.718395128	7.24687E-06	21108	8274	0.71839902	6.885E-06

Table 8. Bayesian analysis for August

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	26	16	0.619047619	0.005484364	1277	265	0.828145266	9.22363E-05
2	1277	265	0.828145266	9.22363E-05	2528	514	0.831032216	4.61445E-05
3	2528	514	0.831032216	4.61445E-05	3779	763	0.832012329	3.07655E-05
4	3779	763	0.832012329	3.07655E-05	5030	1012	0.832505793	2.30746E-05
5	5030	1012	0.832505793	2.30746E-05	6281	1261	0.83280297	1.84598E-05
6	6281	1261	0.83280297	1.84598E-05	7532	1510	0.833001548	1.53832E-05
7	7532	1510	0.833001548	1.53832E-05	8783	1759	0.833143616	1.31856E-05
8	8783	1759	0.833143616	1.31856E-05	10034	2008	0.833250291	1.15373E-05
9	10034	2008	0.833250291	1.15373E-05	11285	2257	0.833333333	1.02554E-05
10	11636	1906	0.859252695	8.92989E-06	12887	2155	0.856734477	8.15931E-06
11	12887	2155	0.856734477	8.15931E-06	14138	2404	0.854672954	7.50814E-06
12	14138	2404	0.854672954	7.50814E-06	15389	2653	0.852954218	6.95136E-06
13	15389	2653	0.852954218	6.95136E-06	16640	2902	0.851499335	6.47026E-06
14	16640	2902	0.851499335	6.47026E-06	17891	3151	0.850251877	6.05064E-06
15	17891	3151	0.850251877	6.05064E-06	19142	3400	0.849170437	5.68159E-06
16	19142	3400	0.849170437	5.68159E-06	20393	3649	0.848223941	5.35458E-06
17	20666	20393	3649	6.08838E-06	21917	20642	0.514979205	5.86879E-06
18	21917	20642	0.514979205	5.86879E-06	23168	20891	0.52584035	5.65893E-06
19	23168	20891	0.52584035	5.65893E-06	24419	21140	0.535986303	5.45885E-06
20	24419	21140	0.535986303	5.45885E-06	25670	21389	0.545485454	5.2684E-06

Table 9. Bayesian analysis for September

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	29	13	0.690476	0.004970205	1207	335	0.782749676	0.000110209
2	1207	335	0.78275	0.000110209	2385	657	0.784023669	5.56459E-05
3	2385	657	0.784024	5.56459E-05	3563	979	0.784456187	3.72187E-05
4	3563	979	0.784456	3.72187E-05	4741	1301	0.784673949	2.79597E-05
5	4741	1301	0.784674	2.79597E-05	5919	1623	0.784805091	2.23898E-05
6	5919	1623	0.784805	2.23898E-05	7097	1945	0.784892723	1.86704E-05
7	7097	1945	0.784893	1.86704E-05	8275	2267	0.784955416	1.60107E-05
8	8275	2267	0.784955	1.60107E-05	9453	2589	0.785002491	1.40142E-05
9	9453	2589	0.785002	1.40142E-05	10631	2911	0.785039137	1.24605E-05
10	10631	2911	0.785039	1.24605E-05	11809	3233	0.785068475	1.12169E-05
11	11809	3233	0.785068	1.12169E-05	12987	3555	0.785092492	1.0199E-05
12	12987	3555	0.785092	1.0199E-05	14165	3877	0.785112515	9.35049E-06
13	14165	3877	0.785113	9.35049E-06	15343	4199	0.785129465	8.63231E-06
14	15343	4199	0.785129	8.63231E-06	16521	4521	0.785143998	8.01658E-06
15	16521	4521	0.785144	8.01658E-06	17699	4843	0.785156597	7.48284E-06
16	17699	4843	0.785157	7.48284E-06	18877	5165	0.785167623	7.01574E-06
17	18877	5165	0.785168	7.01574E-06	20055	5487	0.785177355	6.60353E-06
18	20055	5487	0.785177	6.60353E-06	21233	5809	0.785186007	6.23706E-06
19	21233	5809	0.785186	6.23706E-06	22411	6131	0.78519375	5.90914E-06
20	22411	6131	0.785194	5.90914E-06	23589	6453	0.785200719	5.61397E-06

Table 10. Bayesian analysis for October

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	29	9	0.763157895	0.004634562	1104	434	0.717815345	0.000131616
2	1104	434	0.717815345	0.000131616	2179	859	0.71724819	6.67335E-05
3	2179	859	0.71724819	6.67335E-05	3254	1284	0.717055972	4.46985E-05
4	3254	1284	0.717055972	4.46985E-05	4329	1709	0.716959258	3.3603E-05
5	4329	1709	0.716959258	3.3603E-05	5404	2134	0.716901035	2.69205E-05
6	5404	2134	0.716901035	2.69205E-05	6479	2559	0.716862138	2.2455E-05
7	6479	2559	0.716862138	2.2455E-05	7554	2984	0.716834314	1.92602E-05
8	7554	2984	0.716834314	1.92602E-05	8629	3409	0.716813424	1.68612E-05
9	8629	3409	0.716813424	1.68612E-05	9704	3834	0.716797164	1.49936E-05
10	9704	3834	0.716797164	1.49936E-05	10779	4259	0.716784147	1.34985E-05
11	10779	4259	0.716784147	1.34985E-05	11854	4684	0.716773491	1.22746E-05
12	11854	4684	0.716773491	1.22746E-05	12929	5109	0.716764608	1.12541E-05
13	12929	5109	0.716764608	1.12541E-05	14004	5534	0.716757089	1.03903E-05
14	14004	5534	0.716757089	1.03903E-05	15079	5959	0.716750642	9.64966E-06
15	15079	5959	0.716750642	9.64966E-06	16154	6384	0.716745053	9.00757E-06
16	16154	6384	0.716745053	9.00757E-06	17229	6809	0.716740161	8.4456E-06
17	17229	6809	0.716740161	8.4456E-06	18304	7234	0.716735845	7.94963E-06
18	18304	7234	0.716735845	7.94963E-06	19379	7659	0.716732007	7.50868E-06
19	19379	7659	0.716732007	7.50868E-06	20454	8084	0.716728572	7.11408E-06
20	20454	8084	0.716728572	7.11408E-06	21529	8509	0.716725481	6.75888E-06

Table 11. Bayesian analysis for November

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	28	6	0.823529412	0.004152249	1406	128	0.916558018	4.98237E-05
2	1406	128	0.916558018	4.98237E-05	2784	250	0.917600527	2.49126E-05
3	2784	250	0.917600527	2.49126E-05	4162	372	0.917953242	1.66075E-05
4	4162	372	0.917953242	1.66075E-05	5540	494	0.918130593	1.24551E-05
5	5540	494	0.918130593	1.24551E-05	6918	616	0.918237324	9.96384E-06
6	6918	616	0.918237324	9.96384E-06	8296	738	0.918308612	8.30303E-06
7	7766	1276	0.858880779	1.34031E-05	9144	1398	0.867387592	1.09102E-05
8	9144	1398	0.867387592	1.09102E-05	10522	1520	0.87377512	9.1582E-06
9	10522	1520	0.87377512	9.1582E-06	11900	1642	0.8787476	7.86755E-06
10	11900	1642	0.8787476	7.86755E-06	13278	1764	0.882728361	6.88154E-06
11	13278	1764	0.882728361	6.88154E-06	14656	1886	0.885987184	6.10614E-06
12	14656	1886	0.885987184	6.10614E-06	16034	2008	0.888704135	5.48185E-06
13	16034	2008	0.888704135	5.48185E-06	17412	2130	0.891003991	4.96934E-06
14	17412	2130	0.891003991	4.96934E-06	18790	2252	0.892975953	4.54165E-06
15	18790	2252	0.892975953	4.54165E-06	20168	2374	0.894685476	4.17972E-06
16	20168	2374	0.894685476	4.17972E-06	21546	2496	0.896181682	3.86974E-06
17	21546	2496	0.896181682	3.86974E-06	22924	2618	0.897502153	3.60146E-06
18	22924	2618	0.897502153	3.60146E-06	24302	2740	0.898676133	3.36713E-06
19	24302	2740	0.898676133	3.36713E-06	25680	2862	0.899726719	3.16079E-06
20	25680	2862	0.899726719	3.16079E-06	27058	2984	0.900672392	2.97779E-06

Table 12. Bayesian analysis for December

Iteration	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	27	7	0.794117647	0.00467128	1332	202	0.868318123	7.44897E-05
2	1332	202	0.868318123	7.44897E-05	2637	397	0.869149637	3.74723E-05
3	2637	397	0.869149637	3.74723E-05	3942	592	0.869430966	2.50321E-05
4	3942	592	0.869430966	2.50321E-05	5247	787	0.869572423	1.87931E-05
5	5247	787	0.869572423	1.87931E-05	6552	982	0.869657552	1.50436E-05
6	6552	982	0.869657552	1.50436E-05	7857	1177	0.869714412	1.25414E-05
7	7857	1177	0.869714412	1.25414E-05	9162	1372	0.869755079	1.07528E-05
8	9162	1372	0.869755079	1.07528E-05	10467	1567	0.869785607	9.41077E-06
9	10467	1567	0.869785607	9.41077E-06	11772	1762	0.869809369	8.36653E-06
10	11772	1762	0.869809369	8.36653E-06	13077	1957	0.869828389	7.53089E-06
11	13077	1957	0.869828389	7.53089E-06	14382	2152	0.869843958	6.84702E-06
12	14382	2152	0.869843958	6.84702E-06	15687	2347	0.869856937	6.27701E-06
13	15687	2347	0.869856937	6.27701E-06	16992	2542	0.869867923	5.79461E-06
14	16992	2542	0.869867923	5.79461E-06	18297	2737	0.869877341	5.38107E-06
15	18297	2737	0.869877341	5.38107E-06	19602	2932	0.869885506	5.02262E-06
16	19602	2932	0.869885506	5.02262E-06	20907	3127	0.869892652	4.70894E-06
17	20907	3127	0.869892652	4.70894E-06	22212	3322	0.869898958	4.43214E-06
18	22212	3322	0.869898958	4.43214E-06	23517	3517	0.869904565	4.18608E-06
19	23517	3517	0.869904565	4.18608E-06	24822	3712	0.869909582	3.9659E-06
20	24822	3712	0.869909582	3.9659E-06	26127	3907	0.869914097	3.76772E-06

Table 13. Bayesian analysis for cluster one

ITERATION S	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	23	9	0.71875	0.00612571	1103	429	0.71997389	0.000131514
2	1103	429	0.71997389	0.000131514	2183	624	0.777698611	6.15682E-05
3	2408	624	0.794195251	5.38903E-05	3488	819	0.809844439	3.57466E-05
4	3713	819	0.819285084	3.2662E-05	4793	1014	0.825383158	2.4815E-05
5	5018	1014	0.831896552	2.318E-05	6098	1209	0.83454222	1.88946E-05
6	6323	1209	0.839484865	1.7888E-05	7403	1404	0.840581356	1.52139E-05
7	7628	1404	0.844552702	1.45338E-05	8708	1599	0.844862715	1.27153E-05
8	8933	1599	0.848176984	1.22257E-05	10013	1794	0.848056238	1.09127E-05
9	10238	1794	0.850897606	1.05436E-05	11318	1989	0.850529796	9.55281E-06
10	11543	1989	0.853015075	9.26479E-06	12623	2184	0.852502195	8.4915E-06
11	12848	2184	0.854709952	8.26055E-06	13928	2379	0.854111731	7.64072E-06
12	14153	2379	0.856097266	7.45144E-06	15233	2574	0.855450104	6.9438E-06
13	15458	2574	0.857253771	6.78588E-06	16538	2769	0.856580515	6.36267E-06
14	16763	2769	0.858232644	6.22891E-06	17843	2964	0.857547941	5.87079E-06
15	18068	2964	0.85907189	5.75607E-06	19148	3159	0.85838526	5.44917E-06
16	19373	3159	0.859799396	5.34968E-06	20453	3354	0.859117066	5.08379E-06
17	20678	3354	0.860436085	4.99671E-06	21758	3549	0.859762121	4.76415E-06
18	21983	3549	0.860997963	4.68729E-06	23063	3744	0.860334987	4.4822E-06
19	23288	3744	0.861497484	4.41385E-06	24368	3939	0.86084714	4.23164E-06
20	24593	3939	0.861944483	4.17048E-06	25673	4134	0.861307747	4.00754E-06

Table 14. Bayesian analysis for cluster two

ITERATION S	alpha	beta	prior mean	prior variance	posterior alpha	posterior beta	posterior mean	posterior variance
1	24	9	0.727272727	0.005833738	1171	362	0.763861709	0.000117586
2	1171	362	0.763861709	0.000117586	2318	715	0.764259809	5.93826E-05
3	2318	715	0.764259809	5.93826E-05	3465	1068	0.764394441	3.97211E-05
4	3465	1068	0.764394441	3.97211E-05	4612	1421	0.764462125	2.98409E-05
5	4612	1421	0.764462125	2.98409E-05	5759	1774	0.764502854	2.38968E-05
6	5759	1774	0.764502854	2.38968E-05	6906	2127	0.764530056	1.99274E-05
7	6906	2127	0.764530056	1.99274E-05	8053	2480	0.764549511	1.70888E-05
8	8053	2480	0.764549511	1.70888E-05	9200	2833	0.764564115	1.49581E-05
9	9200	2833	0.764564115	1.49581E-05	10347	3186	0.764575482	1.32998E-05
10	10347	3186	0.764575482	1.32998E-05	11494	3539	0.764584581	1.19725E-05
11	11494	3539	0.764584581	1.19725E-05	12641	3892	0.764592028	1.08861E-05
12	12641	3892	0.764592028	1.08861E-05	13788	4245	0.764598237	9.98047E-06
13	13788	4245	0.764598237	9.98047E-06	14935	4598	0.764603492	9.21393E-06
14	14935	4598	0.764603492	9.21393E-06	16082	4951	0.764607997	8.55675E-06
15	16082	4951	0.764607997	8.55675E-06	17229	5304	0.764611903	7.98707E-06
16	17229	5304	0.764611903	7.98707E-06	18376	5657	0.764615321	7.48851E-06
17	18376	5657	0.764615321	7.48851E-06	19523	6010	0.764618337	7.04853E-06
18	19523	6010	0.764618337	7.04853E-06	20670	6363	0.764621019	6.65738E-06
19	20670	6363	0.764621019	6.65738E-06	21817	6716	0.764623418	6.30737E-06
20	21817	6716	0.764623418	6.30737E-06	22964	7069	0.764625579	5.99232E-06

Table 15. Final iterative values of the Bayesian analysis

MONTH	PRIOR MEAN	PRIOR VARIANCE	POSTERIOR MEAN	POSTERIOR VARIANCE
JANUARY	0.869	3.97E-06	0.899	3.77E-06
FEBRUARY	0.824	6.02E-06	0.824	5.71E-06
MARCH	0.9	3.16E-06	0.9	2.99E-06
APRIL	0.859	4.22E-06	0.859	4.02E-06
MAY	0.923	2.48E-06	0.923	2.35E-06
JUNE	0.769	6.22E-06	0.769	5.96E-06
JULY	0.718	7.25E-06	0.718	6.88E-06
AUGUST	0.536	5.45E-06	0.536	5.27E-06
SEPTEMBER	0.785	5.91E-06	0.785	5.61E-06
OCTOBER	0.717	7.11E-06	0.717	6.76E-06
NOVEMBER	0.899	3.16E-06	0.899	2.98E-06
DECEMBER	0.869	3.97E-06	0.869	3.77E-06
	BAYESIAN SUMMARY OF THE CLUSTERS			
CLUSTER ONE	0.86	4.17048E-06	0.86	4.00754E-06
CUSTER TWO	0.765	6.30737E-06	0.764	5.99232E-06

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