British Journal of Mathematics & Computer Science
16(5): 1-9, 2016, Article no.BJMCS.25746
ISSN: 2231-0851

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Blow-up of Solutions for a Reaction-diffusion Equation with Nonlinear Nonlocal Boundary Condition

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/BJMCS/2016/25746
Editor(s):
(1) Andrej V. Plotnikov, Department of Applied and Calculus Mathematics and CAD, Odessa State Academy of Civil Engineering and Architecture, Ukraine.
Reviewers:
(1) Andrej Kon'kov, Moscow Lomonosov State University, Russia.
(2) Haihua Lu, Nantong University, China.
(3) Xinguang Yang, Henan Normal University, China.
(4) Yongjun Li, Lanzhou City University, China.
Complete Peer review History: http://sciencedomain.org/review-history/14531

Received: 18th March 2016
Accepted: 6th April 2016
Published: 9th May 2016

Original Research Article

Abstract

This paper deals with a reaction-diffusion equation subject to nonlinear nonlocal boundary condition and with nonlocal reaction source. We investigate the global existence and blow up in finite time of a nonnegative solution by using a sub-super solution method.

Keywords: Reaction-diffusion equation; nonlocal source; nonlinear nonlocal boundary condition; blow-up; global solution.

2010 Mathematics Subject Classification: 35B44; 35K58.

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1 Introduction

In this paper we consider the following nonlocal initial boundary value problem

$$u_t = \Delta u + c(x,t)u^p \int_\Omega u^q(y,t)dy, \quad x \in \Omega, 0 < t < T, \tag{1.1}$$

$$\frac{\partial u}{\partial \nu} = \int_\Omega k(x,y,t)u^l(y,t)dy, \quad x \in \partial\Omega, 0 < t < T, \tag{1.2}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \tag{1.3}$$

where $p, q, l > 0$, Ω is a bounded domain in $R^n (n \geq 1)$ with smooth boundary $\partial\Omega$, ν is unit outward normal on $\partial\Omega$. Here $c(x,t)$ is a positive continuous bounded function defined for $x \in \bar{\Omega}$, $t \in [0,T]$ and $k(x,y,t)$ is a positive continuous bounded function defined for $x \in \partial\Omega$, $y \in \Omega$, $t > 0$. Furthermore, we assume that $k(x,\cdot,t) \not\equiv 0$ for any $x \in \partial\Omega$ and $t > 0$. The initial datum $u_0(x) \in C^{2+\alpha}(\Omega) \cap C(\bar{\Omega})$ with $0 < \alpha < 1$, and further, we assume that $u_0(x) \geq 0, \not\equiv 0$ and satisfies the compatibility conditions.

In the past several decades, many physical phenomena were formulated into nonlocal mathematical models(see[1]-[6] and the references therein). There has been a considerable amount of literature dealing with the properties of solutions to local semilinear parabolic equation or systems of heat equations with homogeneous Diriclet boundary conditions or with nonlinear boundary conditions (see [7]-[13] and references therein). However, there are some important phenomena formulated as parabolic equations which are coupled with nonlocal boundary conditions in mathematical modeling such as thermoelasticity theory (see [14]-[19]).

The problem of nonlocal boundary value for linear parabolic equations of the type

$$u_t - Au = c(x)u, \quad x \in \Omega, t > 0, \tag{1.4}$$

$$u(x,t) = \int_\Omega \varphi(x,y)u(y,t)dy, \quad x \in \partial\Omega, t > 0, \tag{1.5}$$

$$u(x,0) = u_0(x), \quad x \in \Omega \tag{1.6}$$

with uniformly elliptic operator $A = \sum_{i,j=1}^n a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j} - \sum_{i=1}^n b_i(x)\frac{\partial}{\partial x_i}$ and $c(x) \leq 0$ was studied by Friedman[20]. The global existence and monotonic decay of the solution of problem (1.4)-(1.6) were obtained under the condition $\int_\Omega |\varphi(x,y)|dy < 1$ for all $x \in \partial\Omega$. And later the problem (1.4)-(1.6) with Au replaced by Δu and the linear term $c(x)u$ replaced by the nonlinear term $g(x,u)$ was discussed by Deng [21]. The comparison principle and the local existence were established. On the basis of Deng's work, Seo in [22] investigated the above problem with $g(x,u) = g(u)$, by using the upper and lower solution's technique, he gained the blow-up condition of positive solution, and in the special case $g(u) = u^p$ or $g(u) = e^u$ he also derived the blow-up rate estimates. In[23], Pao gave the numerical solutions for diffusion equations with nonlocal boundary conditions. Parabolic with both nonlocal sources and nonlocal boundary conditions have been studied as well. For example, the problem of the form

$$u_t - \Delta u = \int_\Omega g(u(y,t))dy,$$

$$Bu = \int_\Omega K(x,y)u(y,t)dy, \quad x \in \partial\Omega, t > 0,$$

$$u(x,0) = u_0(x), \quad x \in \Omega$$

was studied by Lin and Liu [24]. They established local existence, global existence and nonexistence of solutions and discussed the blow-up properties of solutions.

In recent years, the mathematical investigation of blow-up phenomena of solutions to parabolic equation and system subject to nonlinear nonlocal boundary conditions has received a great deal of attention. The following heat equation with nonlinear nonlocal boundary condition

$$u_t = \Delta u + c(x,t)u^p, \quad x \in \Omega, t > 0,$$
$$u(x,t) = \int_\Omega k(x,y,t)u^l(y,t)dy, \quad x \in \partial\Omega, t > 0,$$
$$u(x,0) = u_0(x), \quad x \in \Omega,$$

were studied by Gladkov and Kim in [25]. Comparison principle, the uniqueness of solution with any initial data for $\min(p,l) \geq 1$ and with nontrivial initial data otherwise, nonuniqueness of solution with trivial initial data for $p < 1$ or $l < 1$, local existence theorem had been proved. In [26] they gave some criteria for the existence of global behavior of the coefficients $c(x,t)$ and $k(x,y,t)$ as t tends to infinity. Cui and Yang in [7] investigated

$$u_t = \Delta u + c(x,t)u^p \int_\Omega u^q(y,t)dy, \quad x \in \Omega, t > 0,$$
$$u(x,t) = \int_\Omega k(x,y,t)u^l(y,t)dy, \quad x \in \partial\Omega, t > 0,$$
$$u(x,0) = u_0(x), \quad x \in \Omega,$$

they obtained some criteria which determine whether the solutions blow up in finite time or exist globally, moreover, the blow up rate estimates were also obtained.

In [27] and[28], Gladkov A. and Kavitova T. investigated the following problem

$$u_t = \Delta u + c(x,t)u^p, \quad x \in \Omega, t > 0,$$
$$\frac{\partial u}{\partial \nu} = \int_\Omega k(x,y,t)u^l(y,t)dy, \quad x \in \partial\Omega, t > 0,$$
$$u(x,0) = u_0(x), \quad x \in \Omega.$$

In that two articles, they gave the existence theorem of a local solution and studied the problem of uniqueness and nonuniqueness. Criteria on this problem which determine whether the solutions blow up in finite time for large or for all nontrivial initial data were also given.

Motivated by the above cited works, we aim to establish the global existence and finite time blow up of the solution for problem(1.1)-(1.3).

The plan of this paper is as follows: in the next section, we deals with maximum principle and comparison principle, and give local existence in time for the solution. In Section 3 we establish conditions for global existence and blow-up in finite time.

2 The Comparison Principle and Local Existence

In this section we start with the definition of supersolution and subsolution of problem (1.1)-(1.3). Then we present some material needed in the proof of our main results. For convenience, We set $Q_T = \Omega \times (0,T)$, $S_T = \partial\Omega \times (0,T)$, and $\Gamma_T = S_T \cup \bar{\Omega} \times \{0\}, T > 0$.

Definition 2.1. We say that a nonnegative function $\tilde{u} \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ is a subsolution of (1.1)-(1.3) if

$$\tilde{u}_t \le \Delta\tilde{u} + c(x,t)\tilde{u}^p \int_\Omega \tilde{u}^q dx, \quad (x,t) \in Q_T, \tag{2.1}$$

$$\frac{\partial \tilde{u}(x,t)}{\partial \nu} \le \int_\Omega k(x,y,t)\tilde{u}^l(y,t)dy, \quad (x,t) \in S_T, \tag{2.2}$$

$$\tilde{u}(x,0) \le u_0(x), \quad x \in \Omega. \tag{2.3}$$

A supersolution $\hat{u} \in C^{2,1}(Q_T) \cap C(Q_T \cup \Gamma_T)$ of problem (1.1)-(1.3) is defined analogously by the above inequalities with each inequality reversed. We say that $u(x,t)$ is a solution of the problem (1.1)-(1.3) in Q_T if it is both a subsolution and supersolution of (1.1)-(1.3) in Q_T.

To prove the main results, we use the positiveness of solution.

Lemma 2.1. *Suppose that $u_0 \not\equiv 0$ in Ω and $u(x,t)$ is a solution of (1.1)-(1.3) in Q_T. Then $u(x,t) > 0$ in $Q_T \cup \Gamma_T$.*

Proof. Since $u_0 \not\equiv 0$ and $u_t - \Delta u = c(x,t)u^p \int_\Omega u^q(y,t)dy \ge 0$ in Q_T, by the strong maximum principle $u(x,t) > 0$ in Q_T. If $u(x_0,t_0) = 0$ in some point $(x_0,t_0) \in S_T$, then it yields $\partial u(x_0,t_0)/\partial\nu < 0$, which contradicts (1.2). □

Lemma 2.2. *Let $\hat{u}(x,t)$ and $\tilde{u}(x,t)$ be a supersolution and subsolution of problem (1.1)-(1.3) in Q_T, respectively, with $\hat{u}(x,0) \ge \tilde{u}(x,0)$ and $\hat{u}(x,0) > 0$ in Ω. If $\min(p,q,l) < 1$, we further assume that $\tilde{u}(x,0) > 0$. Then $\hat{u}(x,t) \ge \tilde{u}(x,t)$ in $Q_T \cup \Gamma_T$.*

Proof. Let $\varphi(x,t) \in C^{2,1}(\bar{Q}_t)$ $(0 < t < T)$ be a nonnegative function which satisfies homogeneous Neumann boundary condition. Multiplying (2.1) by φ and then integrating over Q_t, we obtain

$$\begin{aligned}\int_\Omega \tilde{u}(x,t)\varphi(x,t)dx \le &\int_\Omega \tilde{u}(x,0)\varphi(x,0)dx \\ &+ \int_0^t \int_\Omega (\tilde{u}(x,\tau)\varphi_\tau(x,\tau) + \tilde{u}(x,\tau)\Delta\varphi(x,\tau) + c(x,\tau)\varphi(x,\tau)\tilde{u}^p(x,\tau)\int_\Omega \tilde{u}^q(x,\tau))dxd\tau \\ &+ \int_0^t \int_{\partial\Omega} \varphi(x,\tau)\int_\Omega k(x,y,\tau)\tilde{u}^l(y,\tau))dydS_xd\tau.\end{aligned} \tag{2.4}$$

On the other hand, the supersolution $\hat{u}(x,t)$ satisfies

$$\begin{aligned}\int_\Omega \hat{u}(x,t)\varphi(x,t)dx \ge &\int_\Omega \hat{u}(x,0)\varphi(x,0)dx \\ &+ \int_0^t \int_\Omega (\hat{u}(x,\tau)\varphi_\tau(x,\tau) + \hat{u}(x,\tau)\Delta\varphi(x,\tau) + c(x,\tau)\varphi(x,\tau)\hat{u}^p(x,\tau)\int_\Omega \hat{u}^q(y,\tau)dy)dxd\tau \\ &+ \int_0^t \int_{\partial\Omega} \varphi(x,\tau)\int_\Omega k(x,y,\tau)\hat{u}^l(y,\tau))dydS_xd\tau.\end{aligned} \tag{2.5}$$

Put $w(x,t) = \tilde{u} - \hat{u}$, subtracting (2.5) from (2.4) and using mean value theorem, we get

$$\begin{aligned}\int_\Omega w(x,t)\varphi(x,t)dx \le &\int_\Omega w(x,0)\varphi(x,0)dx \\ &+ \int_0^t \int_\Omega w(x,\tau)(\varphi_\tau(x,\tau) + \Delta\varphi(x,\tau) + p\theta_1^{p-1}c(x,\tau)\varphi(x,\tau)\int_\Omega \hat{u}^q(y,\tau)dy)dxd\tau \\ &+ \int_0^t \int_\Omega q\tilde{u}^p c(x,\tau)\varphi(x,\tau)\int_\Omega w(y,\tau)\theta_2^{q-1}(y,\tau)dydxd\tau \\ &+ \int_0^t \int_{\partial\Omega} l\varphi(x,\tau)\int_\Omega k(x,y,\tau)\theta_3^{l-1}(y,\tau)w(y,\tau)dydS_xd\tau,\end{aligned} \tag{2.6}$$

where $\theta_i(x,t)$, $(i = 1, 2, 3)$ are some positive continuous functions in $\bar{Q}_t$ if $\min(p, q, l) < 1$ and some nonnegative continuous functions in $\overline{Q}_t$ otherwise.

The function $\varphi(x,t)$ is defined as a solution of the following problem

$$\varphi_\tau(x,\tau) + \Delta\varphi(x,\tau) + p\theta_1^{p-1}c(x,\tau)\int_\Omega \hat{u}^q(y,\tau)dy\varphi(x,\tau) = 0, \quad (x,\tau) \in Q_t,$$
$$\frac{\partial\varphi}{\partial\nu} = 0, \quad (x,\tau) \in S_t,$$
$$\varphi(x,t) = \psi(x), \quad x \in \Omega,$$

where $\psi(x) \in C_0^\infty(\Omega)$, $0 \le \psi(x) \le 1$. By virtue of the comparison principle for linear parabolic equations the solution $\varphi(x,t)$ is nonnegative and bounded. By (2.6) and $w(x,0) \le 0$ we have

$$\int_\Omega w(x,t)\varphi(x,t)dx \le M\int_0^t\int_\Omega w_+(x,\tau)dxd\tau, \tag{2.7}$$

here we denote $w_+ = \max(0, w)$ and choose

$$M = q\sup_{Q_t}\tilde{u}^p(x,\tau)c(x,\tau)\varphi(x,\tau)\theta_2^{q-1}(x,\tau) + l|\partial\Omega|\sup_{\partial\Omega\times Q_t}k(x,y,\tau)\sup_{Q_t}\theta_3^{l-1}(x,\tau)\sup_{S_t}\varphi(x,\tau).$$

Since the inequality (2.7) holds for every function $\psi(x)$, we can choose a sequence $\psi_n(x) \in C_0^\infty(\Omega)$ converging in $L^1(\Omega)$ to the function

$$\gamma(x) = \begin{cases} 1, & w(x,t) > 0, \\ 0, & w(x,t) \le 0. \end{cases}$$

Substituting $\psi_n(x)$ instead of $\psi(x)$ in (2.7) and letting $n \to \infty$, one have

$$\int_\Omega w_+(x,t)dx \le M\int_0^t\int_\Omega w_+(x,\tau)dxd\tau. \tag{2.8}$$

By using of Gronwall inequality we obtain $w_+(x,t) \le 0$ which implies $\hat{u}(x,t) \ge \tilde{u}(x,t)$ in $Q_T \cup \Gamma_T$. □

Local in time existence of positive classical solutions of problem (1.1)-(1.3) can be obtained by using fixed point theorem, the representation formula and the contraction mapping argument as done in [27], since the proof is more or less standard, we omit it here.

Theorem 2.3. *For some values of T problem (1.1)-(1.3) has maximal solution in Q_T.*

3 Global Existence and Blow-up in Finite Time

Theorem 3.1. *Assume that $p + q \le 1, l \le 1$. Then the solutions of problem (1.1)-(1.3) exist globally for any nonnegative initial data.*

Proof. In order to prove this results, we construct a suitable explicit supersolution of (1.1)-(1.3) in Q_T. Since $c(x,t)$ and $k(x,y,t)$ are continuous functions, there exists a constant M such that $c(x,t) \le M$ and $k(x,y,t) \le M$. Let $\lambda_1$and φ be the first eigenvalue and the corresponding function of the following problem:

$$\Delta\phi + \lambda\phi = 0, \quad x \in \Omega, \tag{3.1}$$
$$\phi(x) = 0, \quad x \in \partial\Omega, \tag{3.2}$$

It is well known that $\phi(x) > 0$ in Ω and $\max_{\partial\Omega} \partial\phi(x)/\partial\nu < 0$. Furthermore, we choose $0 < \varepsilon < 1$ such that

$$\int_\Omega \frac{1}{(\phi(x)+\varepsilon)^q} dx \le 1, \int_\Omega \frac{1}{(\phi(x)+\varepsilon)^l} dx \le 1.$$

Let $\hat{u}(x,t)$ be defined as

$$\hat{u}(x,t) = \frac{d\exp(bt)}{\phi(x)+\varepsilon},$$

where constants d, b will be determined later to guarantee that $\hat{u}$ is a supersolution of problem (1.1)-(1.3). After a direct computation, when $(x,t) \in Q_T$, we have

$$\begin{aligned} \hat{u}_t - &\Delta\hat{u} - c(x,t)\hat{u}^p \int_\Omega \hat{u}^q dx \\ &= b\hat{u} - \left(\frac{\lambda_1}{\phi(x)+\varepsilon} + \frac{2|\nabla\phi|^2}{(\phi(x)+\varepsilon)^2}\right)\hat{u} - c(x,t)\hat{u}^{p+q}(\phi(x)+\varepsilon)^q \int_\Omega \frac{1}{(\phi(x)+\varepsilon)^q} dx \\ &\ge b\hat{u} - \left(\frac{\lambda_1}{\phi(x)+\varepsilon} + \frac{2|\nabla\phi|^2}{(\phi(x)+\varepsilon)^2}\right)\hat{u} - M[\sup_{\overline{\Omega}}(\phi(x)+\varepsilon)]^q \hat{u} \ge 0 \end{aligned} \tag{3.3}$$

if

$$d \ge \sup_{\overline{\Omega}}(\phi(x)+\varepsilon)$$

and

$$b \ge \frac{\lambda_1}{\varepsilon} + \sup_{\overline{\Omega}} \frac{2|\nabla\phi|^2}{(\phi(x)+\varepsilon)^2} + M[\sup_{\overline{\Omega}}(\phi(x)+\varepsilon)]^q.$$

When $(x,t) \in S_T$, we have

$$\begin{aligned} &\frac{\partial\hat{u}}{\partial\nu} - \int_\Omega k(x,y,t)\hat{u}^l(y,t)dy \\ &= \frac{-d\exp(bt)}{(\phi(x)+\varepsilon)^2}\frac{\partial\phi}{\partial\nu} - \int_\Omega k(x,y,t)\left(\frac{d\exp(bt)}{\phi(y)+\varepsilon}\right)^l dy \\ &\ge d\exp(bt)\left(-\frac{\partial\phi}{\partial\nu}\frac{1}{\varepsilon^2} - M\right) \ge 0. \end{aligned} \tag{3.4}$$

if we choose ε is small enough. It is clear from (3.1)-(3.2) that $\hat{u}$ is a supersolution of problem (1.1)-(1.3) in Q_T if $d > \sup_{\overline{\Omega}} u_0(x) + \sup_{\overline{\Omega}}(\phi(x)+\varepsilon)$. Thus the solution of problem (1.1)-(1.3) exists globally. □

Theorem 3.2. *Assume that $p+q<1$, $l>1$. Then the solutions of problem (1.1)-(1.3) exist globally for small initial data.*

Proof. Let $\psi(x)$ be a positive solution of the following problem

$$-\Delta\psi(x) = 1, \quad x \in \Omega, \tag{3.5}$$

$$\frac{\partial\psi}{\partial\nu} = \delta, \quad x \in \partial\Omega. \tag{3.6}$$

We define the function $\hat{u}(x,t) = a\psi(x)$, after simple computation, we get

$$\begin{aligned} &\hat{u}_t - \Delta\hat{u} - c(x,t)\hat{u}^p \int_\Omega \hat{u}^q dx \\ &= -a\Delta\psi - c(x,t)a^{p+q}\psi^p \int_\Omega \psi^q(y)dy \\ &\ge a\left(1 - Ma^{p+q-1}\max_\Omega \psi^{p+q}|\Omega|\right) \ge 0 \end{aligned} \tag{3.7}$$

for large values of a, since $p+q<1$.

On the other hand, we have

$$\frac{\partial \hat{u}}{\partial \nu}=\delta \geq aM(\max_{\Omega}\psi)^l|\Omega| \geq \int_{\Omega} k(x,y,t)\hat{u}^l(y)dy$$

for small values of $\max_{\Omega}\psi$ since $l>1$. Hence $\hat{u}(x,t)$ is a supersolution of (1.1)-(1.3) provided that $u_0(x)\leq M\psi(x)$. By the comparison principle, the problem of (1.1)-(1.3) has global solutions. □

Theorem 3.3. *Assume that* $p+q>1$, $l>0$. *Then the solutions of problem (1.1)-(1.3) blow up in finite time for any positive initial data.*

Proof. Consider the ODE problem

$$\tilde{u}'(t)=m|\Omega|\tilde{u}^{p+q}, \quad \tilde{u}(0)=\tilde{u}_0, \tag{3.8}$$

where m is the lower bound of the functions $c(x,t)$, $k(x,y,t)$. As we all know, the solution to (3.8) blows up in finite time under the assumption $p+q>1$. It is easy to see the solution of (3.8) is a subsolution of problem (1.1)-(1.3) if we choose $\tilde{u}_0=\min_{\overline{\Omega}} u_0(x)$. By the comparison theorem, the solutions of problem (1.1)-(1.3) blow up in finite time. □

Theorem 3.4. *Assume that* $\min(p,q)>1$, $l>0$. *Then the solutions of problem (1.1)-(1.3) blow up in finite time for large initial data.*

Proof. Let ϕ be the eigenfunction of problem (3.1)-(3.2) corresponding to the first eigenvalue λ_1, which is chosen to satisfy that $\int_{\Omega}\phi(x)dx=1$. Now we denote $\phi_s=\sup_{\bar{\Omega}}\phi(x)$, and introduce the following auxiliary function

$$w(t)=\int_{\Omega}u(x,t)\phi(x)dx.$$

Multiplying both sides of the equation of (1.1) by $\phi(x)$ and integrating over Ω, we have

$$w'(t)=\int_{\Omega}\left(\Delta u+c(x,t)u^p\int_{\Omega}u^q dx\right)\phi dx.$$

Then using (3.1),(3.2), Green's identity and the equality $\int_{\partial\Omega}\frac{\partial\phi}{\partial\nu}=-\lambda_1$, we obtain

$$\begin{aligned} w'(t) &= \int_{\Omega}\left(-\lambda_1 u+c(x,t)u^p\int_{\Omega}u^q(y,t)dy\right)\phi(x)dx-\int_{\partial\Omega}u\frac{\partial\phi}{\partial\nu}dS \\ &\geq \int_{\Omega}\left(-\lambda_1 u+c(x,t)u^p\int_{\Omega}u^q(y,t)dy\right)\phi(x)dx \\ &\geq -\int_{\Omega}\lambda_1 u\phi+\frac{m}{\phi_s}\int_{\Omega}u^p\phi dx\int_{\Omega}u^q\phi dx, \end{aligned} \tag{3.9}$$

Further, since $p,q>1$, Jensen's inequality can be applied to (3.9) to get

$$w'(t)\geq -\lambda_1 w(t)+\frac{m}{\phi_s}w^{p+q}(t)$$

with the initial data $w(0)=\int_{\Omega}u_0(x)\varphi(x)dx$. Then if $w(0)>(\frac{\lambda_1\phi_s}{m})^{\frac{1}{p+q-1}}$, by the comparison principle for ordinary equations, we draw the conclusion. □

Acknowledgement

Project Supported by the National Natural Science Foundation of China(No.11571093 and No.11471164).

Competing Interests

The authors declare that no competing interests exist.

References

[1] Anderson JR, Deng K. Global existence for degenerate parabolic equations with a nonlocal forcing. Math. Methods Appl. Sci. 1997;20:1069-1087.

[2] Bebernes JW, Bressan A. Thermal behaviour for a confined reactive gas. J. Differential Equations. 1982;44:118-133.

[3] Bebernes JW, Talaga P. Nonlocal problems modelling shear banding. Nonlinear Anal. 1996;3:79-103.

[4] Furter J, Grinfeld M. Local vs. nonlocal interactions in population dynamics. J. Math. Biol. 1989;29:65-80.

[5] Souplet P. Uniform blow-up profiles and boundary behavior for diffusion equations with nonlocal nonlinear source. J. Differential Equations. 1999;153:374-406.

[6] P. Souplet. Blow-up in nonlocal reaction-diffusion equations. SIAM J. Math. Anal. 1998;29:1301-1334.

[7] Cui ZJ, Yang ZD. Roles of weight functions to a nonlinear porous medium equation with nonlocal source and nonlocal boundary condition. J. Math. Anal. Appl. 2008;342:559-570.

[8] Diaz JI, Kerser R. On a nonlinear degenerate parabolic equation in infiltration or evaporation through a porous medium. J. Differential Equations. 1987;69:368-403.

[9] Deng W, et al. Global existence and nonexistence for a class of degenerate parabolic systems. Nonlinear Anal. 2003;55:233-244.

[10] Deng W, et al. Existence and nonexistence of global solutions of some nonlocal degenerate parabolic equations. Appl. Math. Lett. 2003;16:803-808.

[11] Lin ZG, Xie CH. The blow-up rate for a system of heat equations with nonlinear boundary condition. Nonlinear Anal. 1998;34:767-778.

[12] Li FC, Xie CH. Global existence and blow-up for a nonlinear porous medium equation. Applied Math. Letters. 2003;16:185-192.

[13] Wang MX. Blow-up rate estimates for semilinear parabolic systems. J. Differential Equations. 2001;170:317-324.

[14] Chen YP, Liu LH. Global blow-up for a localized nonlinear parabolic equation with a nonlocal boundary condition. J. Math. Anal. Appl. 2011;384:421-430.

[15] Day WA. A decreasing property of solutions of parabolic equations with applications to thermoelasticity. Quart. Appl. Math. 1983;40:468-475.

[16] Day WA. Extensions of property of heat equation to linear thermoelasticity and other theories. Quart. Appl. Math. 1982;40:319-330.

[17] Pao CV. Numerical solutions of reaction-diffusion equations with nonlocal boundary conditions. J. Comput. Appl. Math. 2001;136:227-243.

[18] Pao CV. Dynamics of reaction-diffusion equations with nonlocal boundary conditions. Quart. Appl. Math. 1995;50:173-186.

[19] Yin YF. On nonlinear parabolic equations with nonlocal boundary condition. J. Math. Anal. Appl. 1994;185:54-60.

[20] Friedman A. Monotonic decay of solutions of Parabolic equations with nonlocal boundary condition. Quart. Appl. Math. 1986;44:401-407.

[21] Deng K. Comparison principle for some nonlocal problems. Quart. Appl. Math. 1992;50:517-522.

[22] Seo S. Blowup of solutions to heat equations with nonlocal boundaryconditions. Kobe. J. Math. 1996;13:123-132.

[23] Pao CV. Asymptotic behavior of solutions of reaction-diffusion equations with nonlocal boundary conditions. J. Comput. Appl. Math. 1998;88:767-778.

[24] Lin ZG, Liu YR. Uniform blow-up profiles for diffusion equations with nonlocal source and nonlocal boundary. Acta Math. Sci. Ser. B. 2004;24:443-450.

[25] Gladkov AL, Kim KI. Uniqueness and nonuniqueness for reaction-diffusion equation with nonlocal boundary condition. Adv. Math. Sci. Appl. 2009;19:39-49.

[26] Gladkov AL, Kim KI. Blow-up of solutions for semilinear heat equation with nonlinear nonlocal boundary condition. J. Math. Anal. Appl. 2008;338:264-273.

[27] Gladkov AL, Kavitova T. Initial boundary value problem for a semilinear parabolic equation with nonlinear nonlocal boundary condition.
Available: http://arxiv.org/abs/1412.5021

[28] Gladkov AL, Kavitova T. Blow-up problem for semilinear heat equation with nonlinear nonlocal boundary condition. Applicable Analysis.
DOI: 10.1080/00036811.2015.1080353

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