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Critical Poses Investigation of Hybrid Parallel Robot with No Identical Legs Based on Double Algebra

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Author's contribution

The sole author designed, analysed, interpreted and prepared the manuscript.

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ABSTRACT

In this paper, the critical poses of PPS-RRS-PRS Hybrid Parallel Robot Manipulators (HPRMs) are geometrically investigated in Double Algebra (DA) approach. The screw theory and a reciprocal screw of dyad joints borrowed from the projective space to obtain the geometrically symbolic form of the inverse of the Jacobian matrix (J) which is expressed in the Global Wrench System (GWS) term called superbrackets. These superbrackets mean the symbolic form of the joints screw lines which are the Plücker coordinate finite lines or lines at infinity related to the Hybrid Parallel Robot Manipulators (HPRMs). The critical configurations arise when these Plücker coordinate lines vectors become linearly dependent at the vanished points of the superbrackets. The results of the investigation are the following: the four planes defined by the position of the joints intersected at last at one point which means that the fourth plane passes through the point defined by the other three. Both the base frame and mobile platform lie in a parallel plan. The key contribution in this paper is a determination of singularity condition of Robots Manipulators and rigidity framework without algebraic calculus by Grassmann-Cayley Algebra approach. This paper calculated the determinant of the Jacobian Matrices in a coordinate- free manner by developing and reducing the Superbracket expression. A novelty of this research from other research is that the Hybrid Parallel

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Robot with no identical leg was cheeked and investigated by Grassmann-Cayley Algebra. Not only fully parallel robot may be studied using Double Algebra, but no-identical legs hybrid parallel robot should be also analysed using Double Algebra.

Keywords: Hybrid robots; double algebra; critical poses; reciprocal screw of dyad joints; superbracket.

1. INTRODUCTION

Hybrid parallel robot manipulators (PRMs) end effector is connected to the base by no-identical legs involves instantaneous motion complexity, such as critical poses problem. At these special configurations, the hybrid parallel robot gains one or more degree of freedom (DOF) and becomes uncontrollable. To avoid this hazardous robot motion which can cause serious damage and injury, many researchers tried to overcome this critical poses problem in the past years. Several of them have suggested a lot of solutions [1-6]. In the most studies of researchers [7-15] critical poses of fully and identical legs of the parallel robot based on Grassmann- Cayley Algebra (or Double Algebra) has been emphasised with attention. Indeed, Double Algebra, also called Grassmann-Cayley Algebra was created by Hermann Grassmann and Arthur Cayley with its two operators, namely join ($\vee$) and meet ($\wedge$) correspond to the geometric operations of summing and intersecting vector respectively in Projective Space. These two operators have dualistic property and can be interchanged [16]. Nevertheless, the studies of the hybrid parallel robot with no-identical legs based on DA or GCA still remain few. The present study aims to investigate the special configurations of no-identical legs P$\underline{\text{P}}$S-R$\underline{\text{R}}$S-P$\underline{\text{R}}$S hybrid parallel robot manipulators. In this paper, the adopted hybrid parallel robot consists of three no-identical legs. The screw theory, its geometric reciprocity and a reciprocal screw of dyad joints for robot manipulators were borrowed from the projective space to obtain the Jacobian matrix (J), which represents Plücker coordinate vector of finite line or line at infinity. Double Algebra approach in projective space enables to determine the symbolic form of the Jacobian Matrix which can be written, developed and reduced in superbracket expression. The critical poses condition that cancels the symbolic form of the determinant of the Jacobian matrix is founded in a coordinate- free manner. The vanished points of the symbolic form of the determinant and their interpretation involve these specials poses analysis of the adopted mechanism. The key contribution of this paper is the determination of singularity condition and frame rigidity of Hybrid Parallels Robots Manipulators without algebraic calculus by Grassmann-Cayley Algebra approach. The paper calculated the determinant of the Jacobian Matrices in a coordinate- free manner by developing and reducing the Superbracket expression. A novelty of this paper, which is different from other researches is that Hybrid Parallel Robot with no identical legs was cheeked and investigated by Grassmann-Cayley Algebra.

The outline of the paper is as follows: section 2 presents screw theory, geometric reciprocal screw, and reciprocal screw of dyad joints for robot manipulators in projective space before the superbracket expression which is a symbolic form of the Jacobian matrix in a free coordinate manner in Double Algebra. Section 3 describes the architecture of the structure and the adopted hybrid parallel robot before the geometric reciprocal screws of dyad joints of the manipulators. Section 4 develops the specials critical poses conditions of the hybrid parallel robot mechanism by analysing and interpreting the vanished condition of previously determined superbracket. Section 5 ends the paper with discussion before conclusion with an overview for future

2. RECIPROCAL SCREWS OF JOINTS AND JACOBIAN MATRIX EXPRESSED IN SUPERBRACKET FORM

Definition of all the terms of abbreviation, symbols and the equations presented in this paper.

Abbreviations		**Descriptions**
GCA	:	Grassmann-Cayley Algebra
GWS	:	Global Wrench System
PRS	:	Prismatic-Revolute-Spherical kinematic chain
PR	:	Dyad Prismatic-Revolute kinematic chain
PS	:	Dyad Prismatic-Spherical kinematic chain

RS : Dyad Revolute-Spherical kinematic chain
$\underline{P}$RS : PRS with actuated Prismatic joint only
P$\underline{R}$S : PRS with actuated Revolute joint only
PR$\underline{S}$: PRS with actuated Spherical joint only
PMs : Parallel Manipulators
SM : Serial Manipulator
ST : Screw Theory
TS : Twist System
TSG : Twist System Graph
WSG : Wrench System Graph
3-PRS : 3 Kinematics chains where each of them consists of Prismatic, Revolute and Spherical joints
6-R : a kinematic chain which consists of six revolute joints.

Symbols: Descriptions

$\vee$: Join operator
$\wedge$: Meet operator
$\bigcap$: Intersection of vectors
$\oplus$: Spanning by vectors
F_i : Constraint wrench force of the i^{th} limb for the Parallel Robot

σ: Permutation
ρ : Wrench intensity
$\Gamma_{p,q}$: Plücker coordinate vector of finite line passing through two distinct finite points p and q

Γ_{∞} : Plücker coordinate vector of lines at infinity, passing through two points at infinity

$J_{6\times 6}$: Jacobian of Square Matrix of six columns six rows
k : Number of joint
l_i : i^{th} Limb or i^{th} kinematic chain
m: Number of link
η : Order of task space
P^3 : Projective Space of 3 dimensional
P_s : Symbolic level of Plücker coordinates
p_i : Prismatic joint axis of i^{th} limb
π_{∞} : Plane at infinity in the Projective Space
r_i : Revolute joint axis of the i^{th} limb
s_i : Spherical joint axis of the i^{th} limb
S_0 : Position Vector of any point on the screw axe
S : Unit vector along the screw axis
$\hat{\$}_0$: Zero pitch screw
$\hat{\$}_{\infty}$: Infinite pitch screw
V : Vector Space

W_n : n^{th} Actuated wrench force

$\hat{\$}_{r0}$: Wrench of zero pitch

$\hat{\$}_{r\infty}$: Wrench of infinite pitch

$[\underbrace{a,\dots,b}_{n-po\,\mathrm{int}\,s}]$: Superbracket which consist of n points

$S_0 \times \mathrm{s}$: Moment of the screw related to the origin of the reference frame.

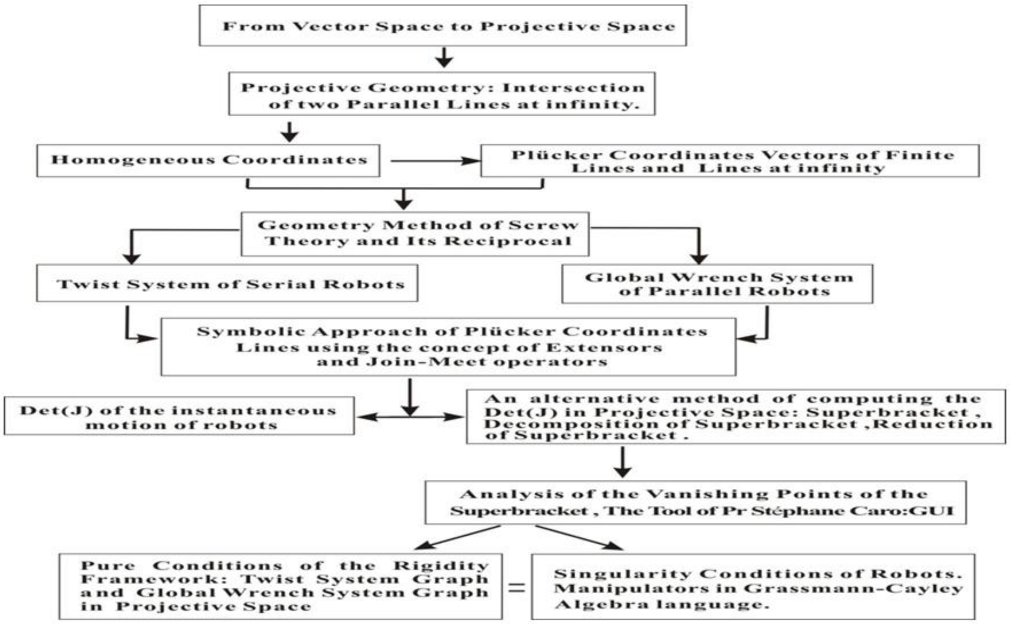

Diagram 1. Frame Work of the Method algorithm in the paper

2.1 Screw Theory and Geometric Reciprocal Screws in Robotic

In projective space, the geometric approach of Plücker coordinates finite lines or lines at infinity have six coordinates where the first three coordinates determine the direction of the line and the last three represents the moment vector of the line [16]. Similarly in kinetic of robot manipulators, six components are either coordinates of the instantaneous motion or the force acting on the robot arm, and then the infinitesimal motion of the robot manipulator can generally be considered as the resulting motion of several instantaneous screws of arbitrary pitches. Instantaneously, a screw motion (Fig. 1) of robot manipulator in a Plücker coordinate line with its associated pitch h is described as:

$$\hat{\$} = [\mathrm{s};(\mathrm{s}_0 \times s + hs]^T \qquad (1)$$

with, s the unit vector along the screw axis, s_0 the position vector of a point on the screw axis with respect to a reference frame, h the pitch of the screw. A zero pitch screw $\hat{\$}_0$ and an infinite pitch screw $\hat{\$}_\infty$ can be respectively identified with a Plücker coordinate vectors of a finite line

$$\hat{\$}_0 = [s;(s_0 \times s)]^T \qquad (2)$$

and a line at infinity

$$\hat{\$}_\infty = [0_{3\times 3}; s]^T \qquad (3)$$

Similarly a wrench is a screw representing a combination of force and couple. When a robot manipulator subjects to a pure force along the axis, ***l*** is a wrench of zero pitch screw (Fig. 2a) and when it subjects to the pure couple is a wrench of infinite pitch screw (Fig. 2b) respectively described as [12]:

$$\hat{\$}_{r0} = [s_r;(s_{r0} \times s_r)]^T \tag{4}$$

$$\hat{\$}_{r\infty} = [0_{3\times3}; s_r]^T \tag{5}$$

The pioneers who investigated the concept of reciprocal screws called these screws as the twists if they represent an instantaneous motion of the rigid body and the wrenches if they represent a system forces and couple acting on a rigid body [5,16]. The main idea of this concept is that if a wrench acts on a robot manipulator in such a way that it produces no work while the robot manipulator undergoes an infinitesimal twist, then both screws representing the twist and the wrench are to be reciprocal to each over. This paper focuses on the geometrical reciprocal screws associated with some dyad screws on robot manipulators (For more details on screws and reciprocal screws see [5,16]). The reciprocal screw associated with a robot manipulator is obtained by an intersection of the systems of reciprocal screw associated with the joints. The dyad joint is just the combination of two joints.

Dyad joints R-S: The joints associated with a revolute-spherical dyad form a four-system. The study deduces that the reciprocal screws form two-system. All reciprocals screws are zero pitches screws forming a planar pencil. They are lines forces passing through the centre of the spherical joint and lie on a plane which contains both the axis of the revolute joint and the centre of the spherical joint as shown in Fig. 3a.

Dyad joints P-S: When it is associated with joints screw, forms four system and with the reciprocal screws form a two-system. According to the reciprocal screw of prismatic and of the spherical screw, the reciprocal screws of dyad joints P-S is defined as the screws passing through the centre of the spherical joint and lie on a plane which is perpendicular to the axis of the prismatic joint as shown in Fig. 3b.

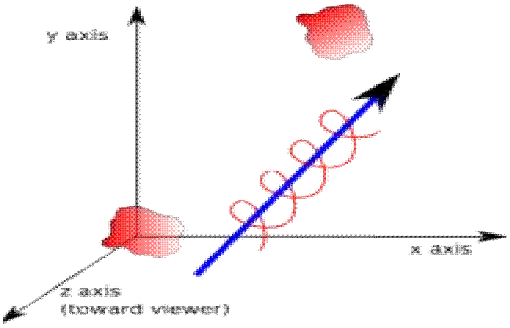

Fig. 1. Screw motion

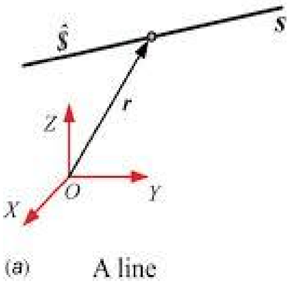

Fig. 2a. Finite line

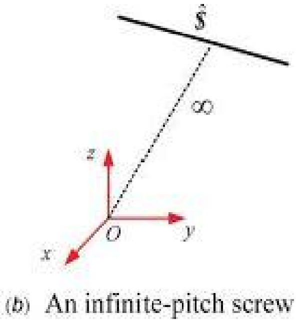

Fig. 2b. Line at infinity

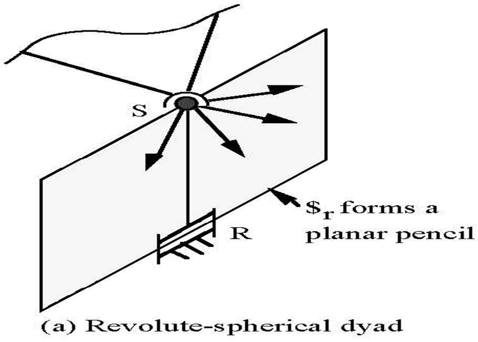

Fig. 3a. R-S Dyad

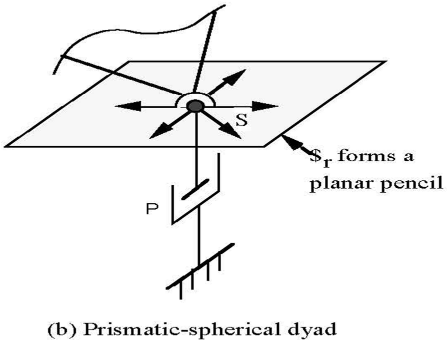

Fig. 3b. P-S Dyad

2.2 Jacobian Matrix of Robot Motion Expressed in Double Algebra Approach

Above determined screw and its reciprocal used to obtain the Jacobian matrix are important to know more about the motion of robot arm. Staffeti and Thomas [16] have shown that the duality is inherent in Double algebra and has been used to reflect the duality between reciprocal twist and wrench which in combination corresponds to the determinant of the Jacobian matrix. The rows of the Jacobian matrix consist of six Plücker vectors of lines or lines at infinity. It's obvious that the determinant of Jacobian matrix becomes zero when the Plücker vectors are linearly dependent at critical poses of the robot arm. The calculation of this determinant is just a superbracket obtained from the joining of wrenches system in Double Algebra approach. These superbrackets can be developed and reduced in the ordinary form. The vanished points and interpretation are the critical poses investigation in Double Algebra. This paper implemented this method on (P$\underline{\text{P}}$S-R$\underline{\text{R}}$S-P$\underline{\text{R}}$S) hybrid parallel manipulators.

3. ARCHITECTURE AND ADOPTED REPRESENTATIONS FOR HYBRID PARALLEL ROBOT

The hybrid parallel robot presents in this work consist of three no-identical kinematics legs. It has three different kinematics legs $\boldsymbol{l_i}$ ***(i=1,2,3)*** with different structures: $\boldsymbol{l_1}$***= P$\underline{\text{P}}$S***; $\boldsymbol{l_2}$***=R$\underline{\text{R}}$S***; $\boldsymbol{l_3}$***=PR$\underline{\text{S}}$***. For l_1 the axis of prismatic joint $P_{1,1}$ is perpendicular to the axis of the prismatic joint $P_{1,2}$ follows a spherical joint at point S_1. For l_2 the axis ofrevolute joint $\boldsymbol{R_{2,1}}$ is perpendicular to $\boldsymbol{R_{2,2}}$, follows a the spherical joint at $\boldsymbol{S_2}$. For l_3 the axis of prismatic joint $P_{3,1}$ is perpendicular to $R_{3,2}$ follows by a spherical joint at S_3. The spherical joint S_i which each consists of three ($\boldsymbol{u_i}$ $\boldsymbol{v_i}$ $\boldsymbol{w_i}$) intersecting non- planar rotation joints at $\boldsymbol{S_i}$ respectively for $\boldsymbol{l_i}$ is connected to the link of the end effector .The moving frame platform centred on ***o'(x',y',z')*** is connected to a fixed base centred on ***o(x,y,z)*** as shown in Fig. 4. Each independent kinematic leg $\boldsymbol{l_i}$ has five degrees of freedom, while the chosen hybrid parallel mechanism has three and described as: the input mechanism adopted in this paper consist of $\boldsymbol{P_{1,2}}$; $\boldsymbol{R_{2,2}}$; $\boldsymbol{R_{3,2}}$ respectively actuated joints of l_1, l_2 and l_3. The screws passing through the centre of the spherical joint S_1 and lie on a plane which is perpendicular to the axis of the prismatic joint $P_{1,1}$ is β_1 as shown in Fig. 5. The screw lines forces passing through the centre of the spherical joint S_2 and lie on a plane which contains both the axis of the revolute joint $R_{2,1}$ and the centre of the spherical joint S_2 is α_2 as shown in Fig. 6. The screws line passing through the centre of the spherical joint S_3 and lie on a plane which is perpendicular to the prismatic joint $P_{3,1}$ is γ_3 as shown in Fig. 7.

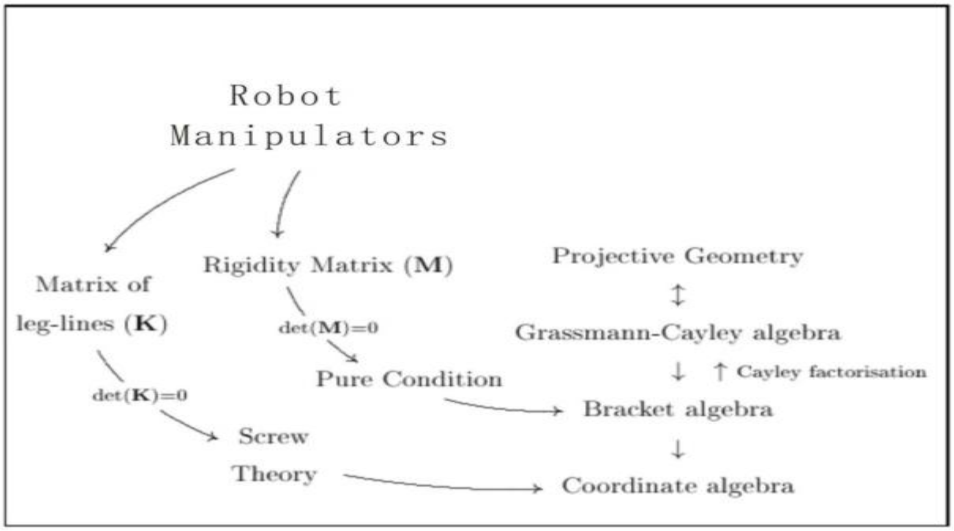

Diagram 2. Analysis of robot critical pose condition based on Grassmann-Cayley approach using bracket as symbolic form of the determinant of Jacobian Matrice

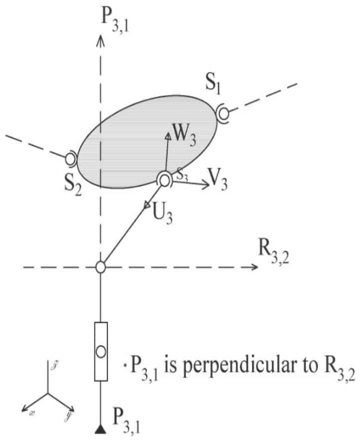

Fig. 4. PRS limb of HPMs

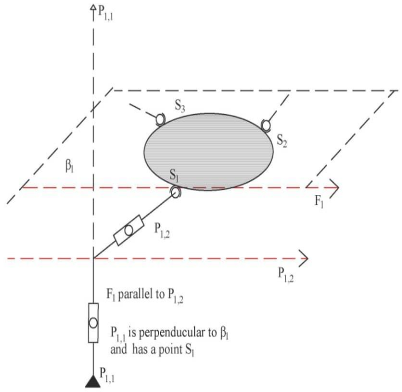

Fig. 5. Reciprocal Wrench β_1

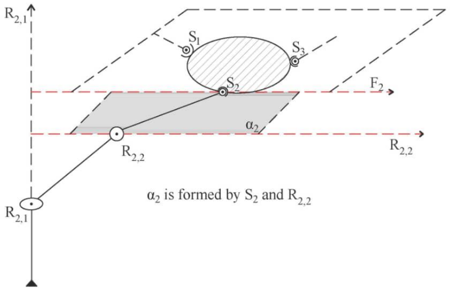

Fig. 6. Reciprocal Wrench System α2

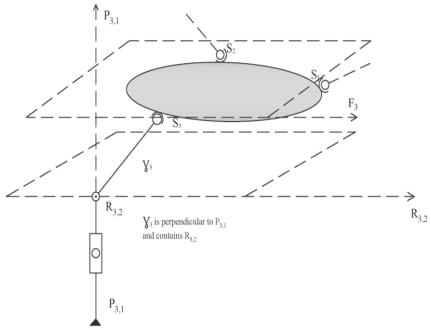

Fig. 7. Reciprocal Wrench System γ_3

4. CRITICAL POSES CONDITION AND INTERPRETATION OF SUPER-BRACKETS VANISHING POINTS USING DOUBLE ALGEBRA

4.1 Analysis of Critical Pose of These no Identical Legs: PPS; RRS and PRS

To know in which condition the hybrid parallel reaches in critical configuration, the Jacobian matrix of the motion of the mechanism must be obtained. The rows of the Jacobian matrix are Plücker coordinate finite lines or line at infinity which provides more information about the architecture structure and constraints force [12-14]. From Double Algebra, the study borrowed some useful tools to express this Jacobian matrix without coordinate. Indeed; the end effector is forced mechanically by these hybrid legs through actuators while the actuators act on the joint screws to reduce the DOF [3] of the mechanism which is described as:

$$DOF = \lambda(n-g-1)+\sum_{i=1}^{g} X_i \qquad (6)$$

$DOF = 6(8-9-1)+(5+5+5)=3$ Where n, g and X_i are respectively the number of bodies, joints and degree of freedom of the $i-th$ joint of the mechanism, therefore,the wrenches system of both constraint wrenches and actuated wrenches system on the hybrid parallel robot should be calculated [16].

4.2 Constraint Wrenches System F$_i$

Since each no-identical leg $\boldsymbol{l_i}$ has 5 serial kinematic chains, each different Twist T^i of a 5-system Twist form respectively 5 different reciprocal constraints wrench of a 1-system F_i to all the 5-system Twist of T^i [12-14]. The constraint wrench system forms a 1-system constraint wrench of zero pitch (Figs. 5 and 6).

1st leg PPS: The corresponding reciprocal constraint is defined as a finite line passing through the centre of a spherical joint S_1 along the direction perpendicular to the prismatic joint $P_{1,1}$ which is as:

$$F_1 = [p_{1,2}, (S_1 \times p_{1,2})] \tag{7}$$

2nd leg RRS: The corresponding reciprocal constraint is defined as a finite line passing through the centre of a spherical joint S_2 along the direction parallel to the revolute joint $\mathbf{R_{2,2}}$ and described as:

$$F_2 = [r_{2,2}, (S_2 \times r_{2,2})] \tag{8}$$

3rd leg PRS: The corresponding reciprocal constraint is defined as a finite line passing through the centre of a spherical joint S_3 along the direction parallel to the revolute joint $\mathbf{R_{3,2}}$ and described as:

$$F_3 = [r_{3,2}, (S_3 \times r_{3,2})] \tag{9}$$

4.3 Actuated Wrenches System W_i

1st leg: PPS: W_1 forms a 2-system wrench only reciprocal to all passive a 4-system Twist $[(P_{1,1})(Su_1)(Sv_1)(Sw_1)]$ because the actuated prismatic joint $P_{1,2}$ is blocked. All reciprocal screws lines lie on plane β_1/ ($F_1 \in \beta_1$)

2nd leg: RRS: W_2 forms a 2-system wrench only reciprocal to all passive a 4-system Twist $[(R_{2,1})(Su_2)(Sv_2)(Sw_2)]$ because the actuated revolute joint $R_{2,2}$ is blocked. All reciprocal screws lines lie on plane α_2 / ($F_2 \in \alpha_2$)

3rd leg: PRS: W_3 forms a 2-system wrench only reciprocal to all passive a 4-system Twist $[(P_{3,1})(Su_3)(Sv_3)(Sw_3)]$ because the actuated prismatic joint $R_{3,2}$ is blocked. All reciprocal screws lines lie on plane γ_3/ ($F_3 \in \gamma_3$)

4.4 Superbrackets

Since a symbolic level of Plücker coordinates without specific coordinate is described as:

$$p_s = w_1 \vee w_2 \vee w_3 \vee \ldots \vee w_k$$
$$p_s = [w_1 w_2 w_3 \ldots w_k]$$

Where the symbol, $\vee$, of join represents the operation of union or joining the associated vector subspaces of two or more extensors.

It is shown that the wrench space of the parallel combination of motion constrains is the sum of the wrenches spaces of the composing constraints [17]. In Double Algebra language the sum of the composing wrenches spaces, applied to a motion for robot manipulator which the centres of motion of the robot legs are joint extensors, is the support of the join of extensors that represent the Global wrench space of the robot manipulator which is also called the superbracket. In this paper, the sum of the wrenches spaces of the composing constraints is:

$$GWS = w_1 \vee w_2 \vee w_3 \vee F_1 \vee F \vee_2 \vee F_3 \tag{10}$$

$$\text{GWS} = [w_1 w_2 w_3 F_1 F_2 F_3] = [\beta_1\ \alpha_2\ \gamma_3\ F_1 F_2 F_3] \tag{11}$$

since $F_1 \in \beta_1$, $F_2 \in \alpha_2$ and $F_3 \in \gamma_3$.

In projective space, any plane may be defined by two different lines and any line is formed by two different points which can be either two different finite points or one finite point and one point at infinity, a point which is not belonging to one line can form a plane with that line. According to the adopted representation in this paper, Double Algebra approach involves the symbolic approach of these six Plücker coordinates finite lines and lines at infinity in projective space is described as:

$$\text{GWS} = [\beta_1\ \alpha_2\ \gamma_3] \tag{12}$$

where $\beta_1=(a,b,n)$; $\alpha_2(ef,gh)$; $\gamma_3=(i,j,m)$ with A= a, N= n, F= f, H= h, I= i and M=m points respectively at infinity (the capital letters are the points at infinity)

$$\text{GWS} = [Ab,bN,eF,gF,Ij,jM] \tag{13}$$

The expression of the superbracket can be developed and reduced in some combinatory of linear monomials brackets. The useful tool, graphic user interface, provided by Stephane Caro performs this computation and gives us the expression below:

[Ab,bN,eF,gF,Ij,jM]= - ([*AbNe*] [bgFj] [FIjM]) + ([AbNF] [bgFj] [eIjM]) + ([AbNg] [beFj] [FIjM]) – ([AbNF] [beFj] [gIjM]) (14)

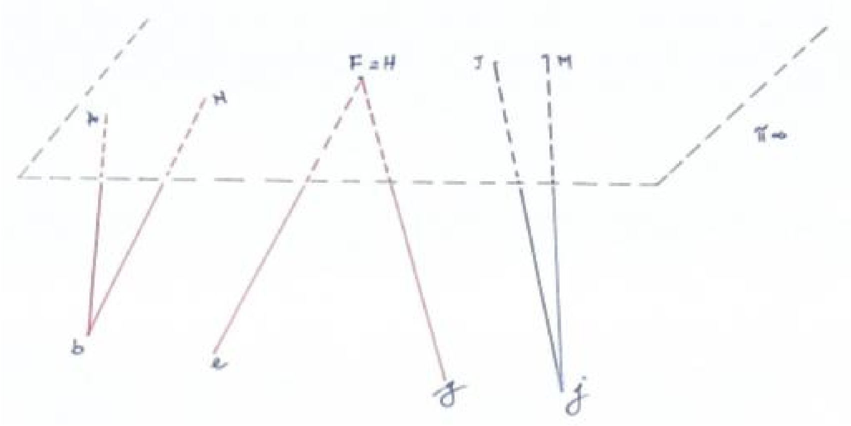

Fig. 8. Wrench graph of the adopted mechanism

Where the vanished points of these superbrackets involve the linearly dependency of Plücker coordinate lines or the critical configurations.

4.5 Wrench Graph for the PPS-RRS-PRS Hybrid Parallel Mechanism

Geometrically, wrench graph is the graphical representation of Global Wrench System which is acting on a PPS-RRS-PRS hybrid parallel robot shown in Fig. 8.

4.6 Interpretation of Vanished Point of the Superbracket

There are neither lines nor tetrahedrons. Therefore fours planes (AbN) (bFj) (IjM) (egF) = 0.For instance,it indicates that the fourth planes passes through the point defined by the other three or the four planes intersect a last at one point .Both the base frame and mobile platform lie in a parallel plan.

5. CONCLUSION

The present approach investigated the critical pose and configurations conditions for hybrid parallel robot manipulator with no identical legs. Indeed Grassmann-Cayley Algebra which is called as Double Algebra in projective space was implemented on, PPS-RRS-PRS, hybrid parallel manipulators. To obtain in coordinate- free manner, the Jacobian matrix was associated to the instantaneous motion of these three no identical legs. Screw theory and geometric reciprocal screw of dyad joint were used to determine respectively the constraint wrench system and the actuated wrench system which in combination obtained the GWS also called the Jacobian matrix or superbrackets in Double algebra language. The vanished points of these superbrackets suggested that the critical pose arose when the four plans intersected a last at one point. The Jacobian matrix six legs lines lie on linear depending or both base and mobile platform in parallel plane. This research aims to implement Double Algebra methodology on hybrid parallel robots, and it was found that critical poses of hybrid manipulators the four planes defined by the position of the joints intersected at last at one point which means the fourth plane passes through the point defined by the other three. Both base frame and mobile platform lie in a parallel plan. Not only fully parallel robot may be studied using Double Algebra, but no identical legs hybrid parallel robot should be also analysed using Double Algebra. The study recognise that the method adopted in the current study does cover neither the varieties (complexities) of hybrid parallel robots nor hybrid robots. The results present in this paper should be useful in the rigidity of the framework for no identical leg at the conceptual stage. Further studies should be focused on the pure condition for either a hybrid parallel robot or hybrid robot.

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COMPETING INTERESTS

Author has declared that no competing interests exist.

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