Constraint and Unconstraint of Vector Autoregressive Model; Using GDP Growth Rate of Agriculture, Industries, Building/Construction, Whole-Sale/Retail and Services in Nigeria

Ekpenyong, Aniedi. Moses[1*] and Isaac, Didi Essi[1]

[1]Department of Mathematics, Rivers State University, Port Harcourt, Nigeria.

Authors' contributions

This work was carried out in collaboration between both authors. Author EAM designed the study, performed the statistical analysis and wrote the protocol, of the manuscript. Author IDE justify the manuscript and over sees the analyses of the study. Both authors read and approved the final manuscript.

Original Research Article

Abstract

In this research, multivariate Time Series was adopted to model the Gross Domestics Product (GDP) growth rate of Nigeria on five (5) variables namely: Agriculture, Industries, Building/construction, Wholesales/Retails trade and Services The data was collected from National Bureau of Statistics, range quarterly from 1985 to 2017, a total of 33years. Real (R) software was used as a tool to analyze the model. The data were grouped into 10 pairs of 2 parameter variables, 10 pairs of 3 parameters variables, 5 pairs of 4 parameters variables, and the complete 5 parameter variables. In each group, the best model was selected and Lag's using Akaike Information Criteria, then the unconstrained (vector autoregressive) AIC of the model was compared with that of constrained (simplified vector autoregressive) AIC model. The unconstraint models with AIC values (-11.973, -17.1111, -22.1823, and 25.8996) at lag (5) was compared with that of constraint models with AIC values of (-12.5116, -17.5298, -22.2894 and -25.9916), the outcome showed that constraint models performed better than unconstraint models.

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Corresponding author: Email: aniekpeyong1@gmail.com;

1 Introduction

Researchers in economics have been interested in forecasting (predicting) data behavior for many years. Numerous method of forecasting has been proposed and developed for this purpose. Using data mining tools and analytical technologies will help to quantify the number of researchers to explore. In any country's economic output, macro-economic variables play critical roles. In the agricultural and non-agricultural sectors, Nigeria's economy has faced various challenges that have contributed to a decline in its growth, which may turn affects, the GDP. Therefore, there is a need to go back to agriculture because of the current unstable economy as a result of the collapse in the oil price on which Nigeria has so much depended in generating its internal revenue (provided 70 percent of Nigeria's IGR). Focusing focus only on the agricultural sector, however, does not solve the problem. The government needs to consider other industries system in making up the Gross Domestic Products of Nigeria (GDP) together with the agricultural sector. Therefore, the objective of this work is to research the interrelationship of Nigeria's GDP between these sectors. Some factors, such as agriculture, industry, wholesale retail, building & services, will be taken into account and also compare the vector autoregressive VAR (p) model with that of the simplified VAR (p) model. This work captures this variable to research its interrelationship with time and build a model to forecast the future of the different sectors under consideration. We will invoke the concept of multivariate time series in modeling.

A variety of debates about the instability of the stock market should not be avoided based on recent economic uncertainty. In certain cases, it is appropriate to evaluate time series using multivariate methods, since univariate analysis could be restrictive. In describing the current market or growth rate of GDP, Robert Engle, a finance professor at New York University stated, "We have no idea where things are going". This simply means high volatility [1].

2 Reviews of Related Work

Mphumuzi [2] looked at Nigerian inflation and economic development. The test was examined using co-integration and Granger Causality. The CPI was used as a proxy for inflation, while GDP was used as a great means for economic growth. For the Nigerian data used, the findings indicated that there was a co-integration relationship between inflation and economic development. Causality that runs from inflation to economic growth also indicates that inflation has an effect on growth, based on empirical findings [3]. In his work titled "Buy-Ballot Modeling of Nigeria Domestic Crude Oil production" applied square root inverse transformation to stabilize its variance. The quadratic pattern was adapted, and it was found that the error variable was random and naturally distributed with a mean zero and some constant variance. Sani and Ismaila [4] calculated a threshold level of inflation for Nigeria using quarterly time-series data and a 13% threshold inflation rate. Its results revealed that inflation had only a small effect on the economic system, while the negative falls of inflation on growth were much greater. Their findings are important for monetary policy formulation because they provide a guide for policymakers to choose an acceptable inflation goal that is compatible with the country's long-term economic growth [5]. Researched "Time Series Modeling with application to South Africa Inflation data". The search was based on financial time series and the ARIMA model. Conditional heteroscedastic (ARCH) models were also fitted to the data [5]. Tools were employed and the adequate model for each family of the model was selected.

Jones [6] researched the co-integration analysis between oil price and consumer price index in South Africa using STATA software. The focus of the paper is to know how substantial inflation is steered by oil prices. The study revealed that global oil price is not the pilot of inflation in south-Africa [7]. Modeled the inflation rate of Ghana from the year 2009-2013 using the Box Jenkins approach. Their result indicated that ARIMA (1,2,1) model was suitable for modeling Ghana's inflation rate with a maximum log-likelihood of -64.21. Ljung-Box test on the residual revealed that the residual is free from the same finite variance (heteroscedasticity) and serial correlation. After verifying stationarity, [8] used autocorrelation and partial autocorrelation to classify the model. The best model among the defined was chosen using the AIC and the SIC. ARIMA (0,1,1) × (0,1,1) was identified as the model that best suit the two series. The forecast was also carried out for crude oil export and

production based on the selected model [9]. In their work, stock trend prediction using regression analysis (A data mining Approach) used regression and a data mining technique and tools were developed to exploit primary time series data in financial institutions. To generate periodic forecasts of stock market prices, they developed a prediction framework that uses data mining techniques. Their approach was used in combination with regression analysis to prove the numeric forecasting method. The input was taken from the financial information acquired from the daily operation summary (equities) released by the Nigerian stock exchange. To explain the patterns of stock market prices on the Nigerian stock exchange, regression analysis was used as a data mining technique. Finally, predictions were made on the future stock market prices of three banks in Nigeria's banking sector. In modeling financial variables including stock returns, [10] used the multivariate system. This multivariate scheme allows stocks to propagate to the others in one variable. To connect vector stationary time series, Campbell used vector autoregression as a mechanism. Furthermore, some financial theories depend on investors' ability to exchange any number of securities without impacting the price. Certain frictions, such as trading prices, short-sale limits, and circuit breakers, affect market creation. Traders, regulators, exchange officials, and researchers have all been drawn to the idea of liquidity. Simply put, liquidity refers to the ability to purchase and sell vast sums of assets easily and at a low cost. [11,12] present theoretical reasons to explain the effect of liquidity on financial market prices. Granger [13] and Amihud [14] found that liquidity predicts expected returns in time series (2002).

As more data became available, the emphasis moved away from researching the series properties of liquidity in equity and fixed income markets [15]. Liquidity risk was compared to planned stock returns. Meanwhile, liquidity was analyzed in the form of transaction costs by [14]. Decisions in financial investment and growth rate of GDP most times rely seriously on the relationship over time among several similar time series. For instance, international currencies exchange rate series. Their contemporaneous variations play important role in the decision relating to the spread of risk in investments [16].

3 Methodology

3.1 Multivariate time series

Multivariate analysis does not limit itself to the later or present of its previous information. Multivariate time series welcomes the past and present information on other series. For instance, the multivariate simplified VAR (p) model, the transfer functions and the intervention models, etc. It's quite clear, that multivariate modeling deals with various considerations of different series. Also, a special case of statistical models that work with dependent variables (factors/ data). So in general, it is farther complicated than that of the univariate model, probably, when the number of series analyzing is more.

3.1.1 Linearity of multivariate Time Series Q_t

Truly speaking multivariate model is nonlinear; moreover linear series can often give an accurate approximation for making a decision.

$$Q_t = \mu + \sum_{i>0}^{\infty} \psi a_{t-i}$$

(3.1)

Q_t Must be within the unit circle

3.1.2 Inevitability of multivariate model

A multivariate series $\{Q_t\}$ is a linear encounter of its lagged values, hence, multivariate time series is always a value of the model Q_t as a tool of its lagged values Q_{t-i} for i is greater than 0 plus new information at time t. This can be presented as

$$Q_t = c + a_t + \sum_{j=1}^{\infty} \pi_j Q_{t-j}$$,
$$\tag{3.2}$$

3.2 Stationary process

A probability process is said to be stationary if its 1st and 2nd moments are dependent on time. That is, a stochastic process Q_t is stationary, if

   i. $E(Q_t) = \mu \ for \ all$ $\tag{3.3}$

   ii. $E\left[(Q_t - \mu)(Q_{t-k} - \mu)'\right] = cov(Q_t \ )$ $\tag{3.4}$

This implies that $\Gamma_z(k) \Rightarrow \Gamma_z(-k)'$ for t and $k = 0,1,2, ...$

3.3 Weak stationarity

An N-dimensional variables Q_t is weakly stationary if

   a. $E\{Q_t\} = \mu, a \ n - dimensional \ vector \ constant$
   b. $cov\{Q_t\} = E\left[(Q_{t-}\mu)(Q_t - \mu)'\right] = \Sigma_{t.......k}$ $\tag{3.5}$

$E\ (Q) and \ Cov\ (Q)$ Present the expectation and covariance matrices of sample vector Q respectively.

3.4 Cross-covariance and correlation matrices

This is a major section in multivariate time series, we cannot do without it. We use this medium to check the linear dynamic dependence of a stationary model Q_t. The classify lag ℓ-cross-covariance matrix as

$$\Gamma_\ell = \mathrm{cov}(Q_t, Q_{t-\ell}) = E[(Q_t - \mu)(Q_{t-\ell} - \mu)^{\mathrm{I}})$$
$$\tag{3.6}$$

3.5 Testing zero cross-covariance

This is the very beginning of the test that is known as the basic test in multivariate time series analysis and it's used in detecting the existence of linear dynamic dependence in the variables. This can be defined as

$$Q_k(m) = T^2 \sum_{\ell=1}^{m} \frac{1}{T-\ell} tr(\widehat{\Gamma}_\ell^{\mathrm{I}} \widehat{\Gamma}_0^{-1} \widehat{\Gamma}_\ell \widehat{\Gamma}_0^{-1})$$
$$\tag{3.7}$$

3.6 Vector Autoregressive (VAR) model

The autoregressive VAR (P) model of the basic P-lag vector is of the form:

$$Q_t = \theta_0 + \ \theta_1 Q_{t-1} + \theta_2 Q_{t-2} + \cdots + \theta_p Q_{t-p} + a_t$$ $\tag{3.8}$

The VAR(P) model can be written in the matrix form as

$$
\begin{pmatrix} Z_{1t} \\ Q_{2t} \\ \vdots \\ Q_{kt} \end{pmatrix} = \begin{pmatrix} \theta_{10} \\ \theta_{20} \\ \vdots \\ \theta_{k0} \end{pmatrix} + \begin{pmatrix} \theta_{11}^1 & \theta_{12}^1 & \dots & \theta_{1k}^1 \\ \theta_{21}^1 & \theta_{22}^1 & \dots & \theta_{2k}^1 \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^1 & \theta_{k2}^1 & \dots & \theta_{kk}^1 \end{pmatrix} \begin{pmatrix} Q_{1t-1} \\ Q_{2t-1} \\ \vdots \\ Q_{kt-1} \end{pmatrix} +
$$

$$
\begin{pmatrix} \theta_{11}^2 & \theta_{12}^2 & \dots & \theta_{1k}^2 \\ \theta_{21}^2 & \theta_{22}^2 & \dots & \theta_{2k}^2 \\ \vdots & & & \vdots \\ \theta_{k1}^2 & \theta_{k2}^2 & \dots & \theta_{kk}^2 \end{pmatrix} \begin{pmatrix} Q_{1t-2} \\ Q_{2t-2} \\ \vdots \\ Q_{kt-2} \end{pmatrix} + \dots +
$$

$$
\begin{pmatrix} \theta_{11}^p & \theta_{12}^p & \dots & \theta_{1k}^p \\ \theta_{21}^p & \theta_{22}^p & \dots & \theta_{2k}^p \\ \vdots & \vdots & & \vdots \\ \theta_{k1}^p & \theta_{k2}^p & \dots & \theta_{kk}^p \end{pmatrix} \begin{pmatrix} Q_{1t-p} \\ Q_{2t-p} \\ \vdots \\ Q_{kt-p} \end{pmatrix} + \begin{pmatrix} a_{1t} \\ a_{2t} \\ \vdots \\ a_{kt} \end{pmatrix}
$$

3.7 Order of selection

In this work, we will emulate the proposed tool of [8], and that consisting of model specification, estimation, and diagnostic checking on multivariate analysis. We will use the recent procedure of [8]. But this approach of selecting the model order of multivariate time series was first proposed by [17]. Behind the approach is to compare different sets of the multivariate model that amount to examining the hypothesis of testing;

$$H_0; \; \emptyset_\rho = 0 \;\; \text{Versus} \; H_\infty; \; \emptyset_\rho \neq 0$$

3.8 Information criteria

We understand that all parameters are based on chance, and they consist of (2) properties. Firstly, the component is concerned with the model data's goodness of fit test, while the second component penalizes more complex models. It is also very true that the consistency of a model's fit test is always calculated by the likelihood maximization. But the maximization probability is centered on the normal distribution corresponding to the determinant of the matrix covariance of the developments, which is also known in multivariate analysis as the generalized variance.

$$AIC(\ell) = In \, | \, \tilde{E}_{a,\ell} \, | + \frac{2}{T} \ell K^2 \tag{3.9}$$

$$BIC(\ell) = In \, | \, \tilde{E}_{a,\ell} \, | + \frac{In(T)}{T} \ell K^2 \tag{3.10}$$

$$HQ(\ell) = In \, | \, \tilde{E}_{a,\ell} \, | + \frac{2In[In(T)]}{T} \ell K^2 \tag{3.11}$$

3.9 Model checking

Model-checking is a major aspect of the examination of the model; it is also known as a diagnostic test. In model design, this plays a significant role, such as multivariate normality, a typical model is said to be adequate.

3.10 Model simplification

If k is moderate or high, multivariate time series models can have a lot of parameters. In practice, we often find that some of the parameters are not statistically relevant at a certain level of significance. The model can then be simplified by eliminating non-essential parameters. This is especially true when no prior knowledge of the

parameters is available. There is, however, no best way to simplify a fitted model. The chi-square test is an alternative to using the knowledge parameters discussed in the criteria to carry out model simplification. We may, for example, estimate the unconstrained VAR (p) model (under Ha) as well as the constrained VAR (p) model (under H0). H0 cannot be rejected based on a chosen criterion if the constrained model has a smaller value for that criterion.

4 Results

4.1 Data

The data for this study came from the National Bureau of Statistics (NBS), and it included quarterly government reports for the GDP growth rates of agriculture, industries, manufacturing, wholesale /retail trade and services from 1985 to 2017, a period of 33 years, and analyzes with Real (R) software.

Table 1. Model selection on 2 parameter variables

S/N	Variables	VAR –Order	AIC	BIC	HQ
1	Agric and Industries	VAR(6)	-8.4799	-7.9558	-8.2669
2	Indust. and B/construct	VAR(5)	-8.9133	-8.4765	-8.358
3	B/const. and W/Retails	VAR(7)	-10.6739	-10.2022	-10.7819
4	W/Retails & Services	VAR(5)	-10.9594	-10.5226	-10.7819
5	Agric and Services	VAR(6)	-10.9454	-10.4334	-10.7324
6	Agric and B/construct	VAR(9)	-10.8197	-10.2599	-10.5618
7	Agric and W/Retails	VAR(8)	-10.2026	-9.7056	-.9.9649
8	Indust. and W/Retails	VAR(5)	-8.1399	-7.5708	-7.9624
9	Indust .and Services	VAR(5)	-9.1725	-8.7357	-8.9950
10	B/const and Services	VAR(5)	-11.8843	-11.4475	-11.7068

On modeling the best model with the least AIC information criteria in Table 1, we obtained the AIC of the simplified (constraint) VAR (5) model to be -12.51167, while -11.97301is the AIC of the unconstrained VAR (5) model for the two-parameter variables.

Table 2. Model selection of 3 parameter variables

S/N	Variables	VAR- Order	AIC	BIC	HQ
1	Agric, Industries, B/construction.	VAR(6)	-14.2395	-13.2402	-13.8236
2	Industries, B/construction W/Retails	VAR(5)	-14.0276	-13.0449	-13.6283
3	B/construction, W/Retails, Services	VAR(5)	-17.0510	-16.0682	-16.6516
4	Agric, Industries. W /Retails	VAR(5)	-13.6047	-12.6214	-13.2049
5	Agric, Industries. and Services	VAR(6)	-14.4149	-13.4028	-13.9862
6	Agric, B/construction, and W/Retails	VAR(5)	-10.8197	-10.2599	-10.5618
7	Agric, B/construction, and Services	VAR(6)	-10.2026	-9.7056	-.9.9649
8	Agric, W/Retails, and Services	VAR(6)	-8.1399	-7.5708	-7.9624
9	Industries, B/construction, and Services	VAR(5)	-9.1725	-8.7357	-8.9950
10	Industries, W/Retails, and Services	VAR(5)	-11.8843	-11.4475	-11.7068

Similarly, we also model the best model of 3 parameter variables in Table 2, the simplified model has-17.5298 as the AIC value of the fitted model while the AIC value of the unconstrained is -17.1111.

Table 3. Model selection of 4 parameter variables

S/N	Variables	VAR –Order	AIC	BIC	HQ
1	Agric, Indust., B/const. W/Retails	VAR(5)	-19.5445	-17.7974	-18.8346
2	Indust., B/const. W/Retails, Serv.	VAR(5)	-20.3677	-18.6205	-19.6577
3	Agric, indust., B/const. Services	VAR(5)	-20.7235	-18.9764	-20.0135
4	Agric, B/const., W /Retails and Services	VAR(5)	-22.0743	-20.9967	-22.0339
5	Agric, Indust. W /Retails and Services	VAR(5)	-19.7718	-18.0246	-19.0618

In the group of 4 parameter variables criteria for model selection, we selected the best model with the least of AIC, BIC, and HQ, and then we model the VAR (5) model and simplified it. The simplified model has-22.2894 AIC value while that of the unconstrained VAR (5) AIC value is -22.1823.

4.2 Order of selecting model on the complete parameter variables

Table 4. Summary table

Lag (p)	AIC	BIC	HQ	M(p)	P-value
0	-12.5550	-12.5550	-12.5550	0.0000	0.0000
1	-22.4248	-21.8788	-22.2029	1152.9597	0.0000
2	-23.0928	-22.0009	-22.6491	112.5352	0.0000
3	-23.9793	-22.3413	-23.3137	129.6844	0.0000
4	-25.0614	-22.8775	-24.1740	142.4419	0.0000
5	-25.8996	-23.4015	-25.0222	134.0173	0.0000
6	-25.7988	-22.7229	-24.6676	21.5344	0.6625
7	-25.7692	-22.0473	-24.3162	20.5621	0.7168
8	-25.7295	-21.4616	-24.0546	26.2778	0.3929

On applying the sequential likelihood ratio test, using three information criteria on the data. We subjected the data into VAR (p) models, in the order; we selected the best model in Table 4 We see that the order selected by AIC, BIC, and HQ of the VAR (5) model, have the least value of AIC, BIC, and HQ. So statistical speaking, the VAR (5) model of the sector's GDP order of selection is the best in modeling the data of Sectors GDP in Nigeria.

However, on comparison of the AIC, the simplified model has-25.99159 which is smaller than the -25.89962 of the unconstrained model.

Table 5. Model summaries

	Variables	Model/ AIC of unconstraint		Simplified(constraint) Model/ AIC	
1	Building &Services	VAR (5)	-11.973		-12.5116
2	B/ Con, W/Ret, and Service	VAR (5)	-17.1111	VAR (5)	-17.5298
3	Agric, B/con, W/Ret and Service	VAR (5)	-22.1823	VAR (5)	-22.2894
4	Agric, Indus, B/con, W/Ret and Service	VAR (5)	-25.8996	VAR (5)	-25.9916

5 Discussion

This work aims to examine the Gross Domestic Products (GDP) growth rate of Nigeria sectors of Agricultures, industries, Building/Construction, Wholesales/ Retails, and Services, from 1985 to 2017. We applied the method of vector autoregressive (VAR) model developed by [18], the order of selection in multivariate time series by [8], the multivariate simplified matrix of [19], modified by [20]. However, we summarized the models in Table 5; from the summary table, we compared the unconstrained VAR (5) model AIC with that of simplified VAR (5) model AIC. From the results, we agreed that the simplified models perform better than that of the unconstrained VAR (5) models. Since the AIC values of the simplified models are less than that of the unconstrained VAR models. Finally, we suggested that it is better to simplify a model by removal the insignificant of the coefficient parameters to obtain good results.

6 Conclusion

In this work, Multivariate Time Series was applied to analyze the model; data were collected from the National Bureau of Statistics, range quarterly from 1985 to 2017, a total of 33 years. To evaluate the model, real (R) software was used. The data were grouped into 10 pairs of 2 parameter variables, 10 combinations of 3

parameters variables, 5 combinations of 4 parameters variables, and the complete 5 parameter variables. In each group, the best model was selected and Lag's using Akaike Information Criteria, in the group of 2 parameters variables in Table 1, the pairs of Building/Construction and Services was selected as the best model with the smallest of AIC (-11.8843) at lag (5) and the plotted graphs indicate a seasonal trend. Building/ Construction, Wholesales/Retails, and Services pairs were selected, as the best model in the group of 3 parameters variables in Table 2, with an AIC value of (-17.0510) and the plotted graphs also indicate a seasonal trend. In a group of 4 parameters variables, the combination of Agriculture, Building/Construction, Wholesales/Retails, and Services in Table 3, was selected as the best model with an AIC value of (-22.7438) as the smallest at lag (5), the plotted graphs also indicate a seasonal trend. In complete parameters variables, Agriculture, Industries, Building/Construction, Wholesales/Retails, and Services, the best lag was selected as lag (5) with the smallest AIC value of (-25.8996), the plotted graphs also indicate seasonal trends. In comparison, the unconstrained (vector autoregressive) AIC of the model was compared with that of the constrained (simplified vector autoregressive) AIC model using Table 5. The unconstraint models with AIC values (-11.973, -17.1111, -22.1823, and 25.8996) at lag (5) was compared with that of constraint models with AIC values of (-12.5116, -17.5298, -22.2894, and -25.9916), the outcome showed that constraint models performed better than unconstraint models in modeling of GDP growth rate of Agricultures, industries, Building/Construction, Wholesales/ Retails, and Services in Nigeria.

Competing Interests

Authors have declared that no competing interests exist.

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