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Research Article

Solutions of Nonlinear Integro-Partial Differential Equations by the Method of $(G'/G, 1/G)$

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In this article, a special expansion method is implemented in solving nonlinear integro-partial differential equations of $(2+1)$-dimensional using a special expansion method of $(G'/G, 1/G)$. We obtained the solutions for $(2+1)$-dimensional nonlinear integro-differential equations in real physical phenomena. The method is applied on $(2+1)$-dimensional space time and solved in three different cases: hyperbolic, trigonometric, and rational functions. The obtained solutions for each result were illustrated by graphical plots using Wolfram Mathematica 9.0 software packages. Furthermore, the obtained results are exactly fit with exact solutions which solves the complicity of finding the solution for nonlinear integro-partial differential equations. Finally, the method is powerful and effective to solve partial differential equations of nonlinear integro form.

1. Introduction

Nonlinear differential equations (NLDEs) are differential equations (DEs) that are not a linear equation in the unknown functions and their derivatives. Many models in mathematics and physics are described by nonlinear partial differential equations (NLPDEs). And they are widely used to describe complex phenomena in various fields of applied sciences, especially in physics and engineering. The investigation of searching solutions for nonlinear integro differential equations plays an important role in nonlinear physical science, because the solutions can describe various natural phenomena of the problems such as wave traveling, vibrations, solitons, and propagation with a finite speed. Different researchers studied the practical and theoretical investigations of applications of partial differential equations [1–5].

Integro-differential equations (IDEs) are differential functional equations involving unknown function $f(x)$ together with both differential and integral operations on f. The integro-partial differential equation (IPDE) is an IDE such that the unknown function depends on more than one independent variables. The IPDE is divided into linear and nonlinear. We model real-life problems usually result in mathematical form as follows: functional equations, ordinary differential equations (ODEs) or partial differential equations (PDEs), integral or IDEs, and stochastic equations. Most mathematical modeling of physical phenomena contains IDEs [6–11]. The theory and application of integro-differential equations play an important role in many fields like engineering sciences, fluid dynamics, and nonlinear optics [12–14].

Ito equation (IE) is one form of NLPDE which describes many branches of physics such as condensed matter physics, fluid dynamics, and optics. The KdV equation was investigated experimentally and theoretically to find its solution [15]. [16] discovered numerically that its solutions seemed to decompose at large times into a collection of solitons: well-separated solitary waves [17]. Ito [18, 19] obtained the well-known generalized $(1+1)$-dimensional and generalized $(2+1)$-dimensional Ito equations by generalization of the bilinear KdV equation. There are two essential types of solutions for NLPDEs, which are analytical (exact solution) and numerical solutions. The methods of solving exact solution of nonlinear integro-partial differential equations (NLIPDEs) are used in differential transform method [20], reliable method [21], Kudryashov method [22], generalized Kudryashov method [23], auxiliary equation method [24, 25], extended auxiliary equation method [26], G'/G-expansion method [27], inverse scattering method [28], modified simple equation method [29–31], $(G'/G, 1/G)$-expansion method [32, 33], and extended simple equation method [34]. NLIPDEs can be observed in various scientific fields, such as plasma physics, optical fibers, fluid dynamics, and chemical physics [35]. The last decades have been witnessed for the discovery of a number of new techniques to solve the nonlinear differential system [36, 37]. Exact solutions for the nonlinear integro-partial differential equations have fundamental importance, since most of the complex phenomena are modeled mathematically by nonlinear integro-partial differential equations.

The nonlinear evolution equations (NLEEs) play a key role in describing a scientific phenomena. Among these, the Korteweg-de Vries (KdV) equation models the shallow water wave dynamics near ocean shore and beaches, the nonlinear Schrodinger's equation describes the dynamics of propagation of solitons through optical fibers, and the Schrodinger-Hirota equation describes the dispersive soliton propagation through optical fibers [35, 38–40]. The investigation of finding exact solution of many kinds of Ito equation has been conducted via different peoples and in different times [41–45].

In some fields such as nuclear reactor dynamics and thermoelasticity, we need to reflect the effect memory of the systems in model. If such systems are modeled using partial differential equations, which involves functions at a given space and time, the effect of past history is ignored. Therefore, in order to incorporate the memory effect in such systems, an integral term in the basic partial differential equation is introduced and this leads to an integro-partial differential equation.

The interaction process of two internal long waves is also described by Ito equation [19] which has the single-soliton solution and periodic solution of Equation (1) by using the Hirota bilinear method. Different scholars have been obtained different solutions. New exact solutions to the $(2+1)$-dimensional Ito equation, extended homoclinic test technique, has been studied [46]. Moreover, [47] constructed the generalized solitary wave solutions by using the Exp-function method. Extended three-wave method for the $(1+2)$-dimensional Ito equation has been studied by [48]. [46] investigated multiperiodic wave solutions of Equation (1) by using Riemann theta function, and [49] discussed the solutions having the nature of breathers, rogue waves, and solitary waves by applying the homoclinic breather limit method. Also, [50] has studied multiple kink solutions and multiple singular kink solutions for $(2+1)$-dimensional nonlinear models generated by the Jaulent-Miodek hierarchy. Very recently, it has also been shown that there are diverse interaction solutions to Equation (1). Finding the exact solution of NLPDEs has advantages better than numerical solutions, since they can reduce the error term.

The Ito equation is the Korteweg-de Vries (KdV) equation type equation and its bilinear transformation. The rolling behavior of ships in the regular sea and interaction process of two internal long waves are often predicted by Ito equation and gave the single-soliton solution and periodic solution of Equation (1) [19]. Generally, in this article, we consider NLPDE of $(2+1)$-dimensional of the following form:

$$W_{tt} + W_{xxxt} + 3(2W_x W_t + W W_{xt}) + 3W_{xx}\int_{\infty}^{x} W_t dx + \alpha W_{yt} + \omega W_{xt} = 0, \tag{1}$$

where $W(x, y, t)$ is an analytic function, $\partial x^{-1} = \int dx$, and α and ω are two auxiliary constants.

If $\alpha = 0$ and $\omega = 0$, then Equation (1) can be reduced to $(1+1)$-dimensional Ito equation which was first proposed by [36].

This article focuses on $(2+1)$-dimensional NLIPD equations when α and ω are different from zero.

Then, by setting $W = u_x$ from Equation (1), we get the following:

$$u_{xtt} + u_{xxxxt} + 3(2u_{xx}u_{xt} + u_x u_{xxt}) + 3u_{xxx}\int_{\infty}^{x} u_{xt} dx + \alpha u_{xyt} + \omega u_{xxt} = 0. \tag{2}$$

Large varieties of physical, chemical, and biological phenomena are governed by NLPDEs. One of the most exciting advances of nonlinear science and theoretical physics has been the development of methods to look for the solutions of NLPDEs. Finding the solutions to NLPDEs plays an important role in nonlinear science like nonlinear wave phenomena of dispersion, dissipation, diffusion, reaction, convection, and generally in nonlinear physical science, since they provide much physical information and more insight into the physical aspects of the problem and thus lead to further applications. Some NLPDE are Ito equation, Sawada-Kotera equation, Burgers-Fisher equation, KP-hierarchy equation, Jaulent–Miodek equation, etc. In this paper, we aim to solve NLPDE using the special expansion method and to illustrate the nature of the obtained solutions in geometrical models.

2. Mathematical Formulation and Results

2.1. Special Expansion Method. In this section, we explain the steps to be followed for the methods of $(G'/G, 1/G)$ which is to determine the solutions of travelling waves of nonlinear physical phenomena. To apply the method of $(G'/G, 1/G)$, we describe the main steps based on [32] as follows:

Remark 1. Let the second-order linear ordinary differential equations (ODEs) of the form

$$G''(\xi) + \lambda G(\xi) = \eta. \tag{3}$$

Set $\phi = G'/G$ and $\beta = 1/G$, and then, we get

$$\phi' = -\phi^2 + \eta\beta - \lambda, \beta' = -\phi\beta. \tag{4}$$

Remark 2. If $\lambda < 0$, then the general solution of Equation (3) has the following form:

$$G(\xi) = A_1 \sin h\left(\xi\sqrt{-\lambda}\right) + A_2 \cos h\left(\xi\sqrt{-\lambda}\right) + \frac{\eta}{\lambda}, \quad (5)$$

where A_1 and A_2 are arbitrary constants.

Consequently, we have

$$\beta^2 = \frac{-\phi^2\lambda + 2\eta\beta\lambda - \lambda^2}{\lambda^2\sigma + \eta^2}, \quad (6)$$

where $\sigma = A_1^2 - A_2^2$.

Remark 3. If $\lambda > 0$, then the general solution of Equation (3) has the following form:

$$G(\xi) = A_1 \sin\left(\xi\sqrt{\lambda}\right) + A_2 \cos\left(\xi\sqrt{\lambda}\right) + \frac{\eta}{2}. \quad (7)$$

Remark 4. If $\lambda = 0$, then the general solution of Equation (3) has the following form: $G'' + \lambda G = \eta$ which becomes $G'' = \eta$. Then, integrating both sides with respect to ξ, it becomes the following: $G' = \eta\xi + A_1$, where A_1 is constant of integration.

Again integrating with respect to ξ, we get the following:

$$G(\xi) = \frac{\eta}{2}\xi^2 + A_1\xi + A_2, \quad (8)$$

where A_1 and A_2 are constants.

And subsequently, it establishes the relation between ϕ and β.

2.1.1. The Main Steps of Special Expansion Method. In this section, we describe the main steps of the special expansion methods by using two variables $(G'/G, 1/G)$ expansion method.

Step 1. First, we change the given nonlinear integro-partial differential equation (NLIPDE) to nonlinear partial differential equations (NLPDEs).

Suppose we have the following NLPDEs in the form

$$G\left(u, u_t, u_x, u_y, u_{xx}, u_{yy}, u_{xt}, u_{xy}, \cdots\right) = 0, \quad (9)$$

where G is a polynomial of unknown function u and its total derivatives with respect to ξ. And its various partial derivatives in which the highest order derivatives and nonlinear terms are involved.

Step 2. We use wave transformation to convert NLPDEs to ODEs:

$$u(x, y, t) = U(\xi), \xi = x + y - kt, \quad (10)$$

where ω is arbitrary constant.

Apply the traveling wave transformation Equation (10) and integrate the resulting equation with respect to ξ as many as possible. And hence, Equation (9) is reduced to the following ODEs:

$$P\left(U, U', U'', U''', U'''', \cdots\right) = 0. \quad (11)$$

Step 3. Assume that the solution of Equation (11) can be expressed by a polynomial in the two variables ϕ and β as follows:

$$U(\xi) = \sum_{i=0}^{N}\left(c_i\phi^i\right) + \sum_{i=1}^{N}\left(b_i\phi^{i-1}\beta\right), \quad (12)$$

where $c_i (i = 0, 1, 2, 3, \cdots N)$ and $b_i (i = 1, 2, 3, \cdots, N)$ are arbitrary constants. And ϕ and β satisfy the following condition.

$$\phi(\xi) = \frac{G'(\xi)}{G(\xi)} \text{ and } \beta(\xi) = \frac{1}{G}(\xi). \quad (13)$$

Step 4. Determine the positive integer N in Equation (12) by using the homogeneous balance equation between the highest order derivatives and the nonlinear term appearing in Equation (11). More precisely, if the degree of $U(\xi)$ is deg $[U(\xi)] = N$, then the degree of other term will be expressed as follows [51]:

$$\deg\left[\frac{d^q U(\xi)}{d\xi^q}\right] = N + q and \deg\left[(U(\xi))^p\left(\frac{d^r U(\xi)}{d\xi^r}\right)^s\right] = Np + s(N + r), \quad (14)$$

where N is the degree of the highest order of the function, q is the order of the highest order, p is the degree of coefficient of nonlinear term, s is the degree of nonlinear term, and r is the order of nonlinear term which can be expressed as follows:

$$N + q = Np + s(N + r). \quad (15)$$

Step 5. From the above steps, we get the value of N and applying Equation (12), we obtain the form of the solution of Equation (11). Then, substitute Equation (12) into Equation (11) along with Equation (4); the left-hand side Equation (11) can be converted into a polynomial in ϕ and β in which the degree of β is not longer than 1. Equating each coefficients of this polynomial to zero yields a system of algebraic equation which can be solved by using the Wolfram Mathematica 9.0 software package to get the values c_i, b_i, k, and η. Proceeding to these steps, we will concentrate on the following applications.

2.2. Application of Special Expansion Method on $(2+1)$-Dimensional Ito Equation. From the traveling wave transformation of Equation (10), we have the following:

$$u(x, y, t) = U(\xi), \xi = x + y - kt. \quad (16)$$

Inserting Equation (10) into Equation (2), we get the following ODE form:

$$(k-\alpha-\omega)U' - U^{(3)} - 3\left(U'\right)^2 + d = 0, \tag{17}$$

where d is constant of integration.

Let setting $U' = V$; then, Equation (17) becomes

$$(k-\alpha-\omega)V - V'' - 3V^2 + d = 0. \tag{18}$$

Balancing the highest order derivative V'' with the nonlinear term V^2 of Equation (18) by Equation (14), we get the following:

$$N + 2 = N \times 0 + 2 \times (N + 0), \tag{19}$$

and we get $N = 2$.

Consequently, the solution of Equation (18) has the following form:

$$V(\xi) = \sum_{i=0}^{2}\left(c_i\phi^i\right) + \sum_{i=1}^{2}\left(b_i\phi^{i-1}\beta\right) = c_o + c_1\phi + b_1\beta + c_2\phi^2 + b_2\phi\beta, \tag{20}$$

where c_0, c_1, c_2, b_1, b_2 are constants.

To solve for Equation (18), first we have to find V, V', V'', and V^2 from Equation (20). We have $V(\xi) = c_o + c_1\phi + b_1\beta + c_2\phi^2 + b_2\phi\beta$. Now, by using Equation (4), we obtain

$$\begin{aligned} V' = & -c_1\phi^2 + c_1\eta\beta - c_1\lambda - b_1\phi\beta - 2c_2\phi^3 + 2c_2\phi\eta\beta \\ & - 2c_2\phi\lambda - b_2\beta\phi^2 + b_2\eta\beta^2 - b_2\lambda\beta + b_2\phi^2\beta. \end{aligned} \tag{21}$$

Differentiating Equation (21), we get the following:

$$\begin{aligned} V'' = & -2c_1\phi\left(-\phi^2 + \eta\beta - \lambda\right) + c_1\eta(-\phi\beta) \\ & - b_1\beta\left(-\phi^2 + \eta\beta - \lambda\right) - b_1\phi(-\phi\beta) \\ & - 6c_2\phi^2\left(-\phi^2 + \eta\beta - \lambda\right) + 2c_2\eta\beta\left(-\phi^2 + \eta\beta - \lambda\right) \\ & + 2c_2\eta\phi(-\phi\beta) - 2c_2\lambda\left(-\phi^2 + \eta\beta - \lambda\right) \\ & - 2b_2\beta\phi\left(-\phi^2 + \eta\beta - \lambda\right) - b_2\phi^2(-\phi\beta) \\ & + 2b_2\eta\beta(-\phi\beta) - b_2\lambda(-\phi\beta) \\ & + 2b_2\beta\phi\left(-\phi^2 + \eta\beta - \lambda\right) + b_2\phi^2(-\phi\beta). \end{aligned} \tag{22}$$

Hence, we get

$$\begin{aligned} V'' = & 2c_1\phi^3 - 3c_1\eta\beta\phi + 2c_1\lambda\phi + 2b_1\beta\phi^2 - b_1\beta^2\eta \\ & + b_1\beta\lambda + 6c_2\phi^4 - 10c_2\phi^2\eta\beta + 8c_2\phi^2\lambda \\ & + 2c_2\beta^2\eta^2 - 4c_2\eta\beta\lambda + 2c_2\lambda^2 - 2b_2\phi\eta\beta^2 + b_2\beta\phi\lambda. \end{aligned} \tag{23}$$

From Equation (20),

$$\begin{aligned} V^2 = & b_1^2\beta^2 + 2\phi b_1 b_2\beta^2 + \phi^2 b_2^2\beta^2 + 2\beta b_1 c_0 + 2\beta\phi b_2 c_0 \\ & + c_0^2 + 2\beta\phi b_1 c_1 + 2\beta\phi^2 b_2 c_1 + 2\phi c_0 c_1 + \phi^2 c_1^2 \\ & + 2\beta\phi^2 b_1 c_2 + 2\beta\phi^3 b_2 c_2 + 2\phi^2 c_0 c_2 + 2\phi^3 c_1 c_2 + \phi^4 c_2^2. \end{aligned} \tag{24}$$

2.2.1. Case I: If $\lambda < 0$ (Hyperbolic Function Solutions). Substituting Equation (6) into Equation (23), we get the following:

$$\begin{aligned} V'' = & 2c_1\phi^3 - 3c_1\eta\beta\phi + 2c_1\lambda\phi + 2b_1\beta\phi^2 \\ & - b_1\eta\left(\frac{-\phi^2\lambda + 2\eta\beta\lambda - \lambda^2}{\lambda^2\sigma + \eta^2}\right) + b_1\beta\lambda + 6c_2\phi^4 \\ & - 10c_2\phi^2\eta\beta + 8c_2\phi^2\lambda + 2c_2\eta^2\left(\frac{-\phi^2\lambda + 2\eta\beta\lambda - \lambda^2}{\lambda^2\sigma + \eta^2}\right) \\ & - 2c_2\eta\beta\lambda + 2c_2\lambda^2 - 2b_2\phi\eta\left(\frac{-\phi^2\lambda + 2\eta\beta\lambda - \lambda^2}{\lambda^2\sigma + \eta^2}\right) \\ & + b_2\beta\phi\lambda = 2c_1\phi^3 - 3c_1\eta\beta\phi + 2c_1\lambda\phi + 2b_1\beta\phi^2 \\ & + \frac{b_1\eta\phi^2\lambda}{\lambda^2\sigma + \eta^2} - \frac{2b_1\eta^2\beta\lambda}{\lambda^2\sigma + \eta^2} + \frac{b_1\eta\lambda^2}{\lambda^2\sigma + \eta^2} \\ & + b_1\beta\lambda + 6c_2\phi^4 - 10c_2\phi^2\eta\beta + 8c_2\phi^2\lambda \\ & - \frac{2c_2\eta^2\phi^2\lambda}{\lambda^2\sigma + \eta^2} + \frac{4c_2\eta^3\beta\lambda}{\lambda^2\sigma + \eta^2} - \frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma + \eta^2} \\ & - 2c_2\eta\beta\lambda + 2c_2\lambda^2 + \frac{2b_2\eta\phi^3\lambda}{\lambda^2\sigma + \eta^2} \\ & - \frac{4b_2\phi\eta^2\beta\lambda}{\lambda^2\sigma + \eta^2} + \frac{2b_2\phi\eta\lambda^2}{\lambda^2\sigma + \eta^2} + b_2\beta\phi\lambda. \end{aligned} \tag{25}$$

Again, putting Equation (6) into Equation (24), we obtain the following:

$$\begin{aligned} V^2 = & -\frac{b_1^2\phi^2\lambda}{\lambda^2\sigma + \eta^2} + \frac{2b_1^2\eta\beta\lambda}{\lambda^2\sigma + \eta^2} - \frac{b_1^2\lambda^2}{\lambda^2\sigma + \eta^2} - \frac{2b_1 b_2\phi^3\lambda}{\lambda^2\sigma + \eta^2} \\ & + \frac{4\phi b_1 b_2\eta\beta\lambda}{\lambda^2\sigma + \eta^2} - \frac{2b_1 b_2\phi\lambda^2}{\lambda^2\sigma + \eta^2} - \frac{\phi^4 b_2^2\lambda}{\lambda^2\sigma + \eta^2} \\ & + \frac{2\phi^2 b_2^2\eta\beta\lambda}{\lambda^2\sigma + \eta^2} - \frac{\phi^2 b_2^2\lambda^2}{\lambda^2\sigma + \eta^2} + 2\beta b_1 c_0 + 2\beta\phi b_2 c_0 \\ & + c_0^2 + 2\beta\phi b_1 c_1 + 2\beta\phi^2 b_2 c_1 + 2\phi c_0 c_1 + \phi^2 c_1^2 \\ & + 2\beta\phi^2 b_1 c_2 + 2\beta\phi^3 b_2 c_2 + 2\phi^2 c_0 c_2 + 2\phi^3 c_1 c_2 + \phi^4 c_2^2. \end{aligned} \tag{26}$$

By using Equations (20), (25), and (26), we obtain Equation (18) equals to

$$
\begin{aligned}
&c_o(k-\alpha-\omega)+c_1\phi(k-\alpha-\omega)+b_1\beta(k-\alpha-\omega)+c_2\phi^2(k-\alpha-\omega)\\
&\quad+b_2\phi\beta(k-\alpha-\omega)-2c_1\phi^3+3c_1\eta\beta\phi-2c_1\lambda\phi-2b_1\beta\phi^2\\
&\quad-\frac{b_1\eta\phi^2\lambda}{\lambda^2\sigma+\eta^2}+\frac{2b_1\eta^2\beta\lambda}{\lambda^2\sigma+\eta^2}-\frac{b_1\eta\lambda^2}{\lambda^2\sigma+\eta^2}-b_1\beta\lambda-6c_2\phi^4\\
&\quad+10c_2\phi^2\eta\beta-8c_2\phi^2\lambda-\frac{2c_2\eta^2\phi^2\lambda}{\lambda^2\sigma+\eta^2}-\frac{4c_2\eta^3\beta\lambda}{\lambda^2\sigma+\eta^2}+\frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma+\eta^2}\\
&\quad+2c_2\eta\beta\lambda-2c_2\lambda^2-\frac{2b_2\eta\phi^3\lambda}{\lambda^2\sigma+\eta^2}+\frac{4b_2\phi\eta^2\beta\lambda}{\lambda^2\sigma+\eta^2}-\frac{2b_2\phi\eta\lambda^2}{\lambda^2\sigma+\eta^2}\\
&\quad-b_2\beta\phi\lambda+\frac{3b_1^2\phi^2\lambda}{\lambda^2\sigma+\eta^2}-\frac{6b_1^2\eta\beta\lambda}{\lambda^2\sigma+\eta^2}+\frac{3b_1^2\lambda^2}{\lambda^2\sigma+\eta^2}\\
&\quad+\frac{6b_1b_2\phi^3\lambda}{\lambda^2\sigma+\eta^2}-\frac{12\phi b_1b_2\eta\beta\lambda}{\lambda^2\sigma+\eta^2}+\frac{6b_1b_2\phi\lambda^2}{\lambda^2\sigma+\eta^2}\\
&\quad+\frac{3\phi^4b_2^2\lambda}{\lambda^2\sigma+\eta^2}-\frac{6\phi^2b_2^2\eta\beta\lambda}{\lambda^2\sigma+\eta^2}+\frac{3\phi^2b_2^2\lambda^2}{\lambda^2\sigma+\eta^2}-6\beta b_1c_0\\
&\quad-6\beta\phi b_2c_0-3c_0^2-6\beta\phi b_1c_1-6\beta\phi^2b_2c_1-6\phi c_0c_1-3\phi^2c_1^2\\
&\quad-6\beta\phi^2b_1c_2-6\beta\phi^3b_2c_2-6\phi^2c_0c_2-6\phi^3c_1c_2-3\phi^4c_2^2+d=0.
\end{aligned}\tag{27}
$$

Setting the coefficients of polynomial ϕ and β zero from Equation (27), we obtain the following sets of algebraic equations:

$$
\begin{aligned}
&\phi^4: -6c_2-3c_2^2+\frac{3b_2^2\lambda}{\lambda^2\sigma+\eta^2}=0,\\
&\phi^3\beta: -6b_2c_2=0,\\
&\phi^3: -2c_2-6c_2c_1-\frac{2b_2\eta\lambda}{\lambda^2\sigma+\eta^2}+\frac{6b_1b_2\lambda}{\lambda^2\sigma+\eta^2}=0,\\
&\phi^2\beta: -2b_1+10c_2\eta-6b_2c_1-6c_2b_1-\frac{6b_2^2\eta\lambda}{\lambda^2\sigma+\eta^2}=0,\\
&\phi^2: c_2(k-\alpha-\omega)-8c_2\lambda-3c_1^2-6c_0c_2-\frac{2c_2\eta^2\lambda}{\lambda^2\sigma+\eta^2}\\
&\qquad+\frac{3b_1^2\lambda}{\lambda^2\sigma+\eta^2}+\frac{3b_2^2\lambda^2}{\lambda^2\sigma+\eta^2}-\frac{b_1\eta\lambda}{\lambda^2\sigma+\eta^2}=0,\\
&\phi\beta: b_2(k-\alpha-\omega)+3c_1\eta-b_2\lambda-6b_2c_0\\
&\qquad-6c_1b_1+\frac{4b_2\eta^2\lambda}{\lambda^2\sigma+\eta^2}-\frac{12b_2b_1\eta\lambda}{\lambda^2\sigma+\eta^2}=0,\\
&\phi: c_1(k-\alpha-\omega)-2c_1\lambda-6c_0c_1-\frac{2b_2\eta\lambda^2}{\lambda^2\sigma+\eta^2}+\frac{6b_1b_2\lambda^2}{\lambda^2\sigma+\eta^2},\\
&\beta: b_1(k-\alpha-\omega)-b_1\lambda-6b_1c_0+2c_2\eta\lambda\\
&\qquad+\frac{2b_1\eta^2\lambda}{\lambda^2\sigma+\eta^2}-\frac{4\eta^3c_2\lambda}{\lambda^2\sigma+\eta^2}-\frac{6\eta\lambda b_1^2}{\lambda^2\sigma+\eta^2},\\
&\phi^0: c_0(k-\alpha-\omega)+2c_2\lambda^2-3c_0^2-\frac{b_1\eta\lambda^2}{\lambda^2\sigma+\eta^2}\\
&\qquad+\frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma+\eta^2}+\frac{3b_1^2\lambda^2}{\lambda^2\sigma+\eta^2}+d=0.
\end{aligned}\tag{28}
$$

Now, by solving the above algebraic equation using Wolfram Mathematica 9.0 software packages, we get the following sets of results.

Result 1

$$
\begin{aligned}
&c_0=\frac{1}{6}\left(-13\lambda\pm\sqrt{97\lambda^2-12d}\right), c_1=0, c_2=-2,\\
&b_1=\pm i\sqrt{10}\sqrt{\sigma}\lambda=2\eta, b_2=0,\\
&k=\alpha\pm\sqrt{97\lambda^2-12d}+\omega, \eta=\pm i\sqrt{\frac{5}{2}}\sqrt{\sigma}\lambda.
\end{aligned}\tag{29}
$$

Now by using the above values of variables and putting into Equation (20), we get the following:

$$
V_{11}=\frac{1}{6}\left(-13\lambda\pm\sqrt{97\lambda^2-12d}\right)+2\eta\beta-2\phi^2.\tag{30}
$$

Then, by using Equation (13), the solution of Equation (2) is as follows:

$$
V_{11}=\frac{1}{6}\left(-13\lambda\pm\sqrt{97\lambda^2-12d}\right)+2\eta\left(\frac{1}{G}\right)-2\left(\frac{G'}{G}\right)^2.\tag{31}
$$

But, when we put the value of G from Equation (5), we get the following:

$$
\begin{aligned}
V_{11}&=\frac{1}{6}\left(-13\lambda\pm\sqrt{97\lambda^2-12d}\right)\\
&\quad+\left(\frac{2\eta}{A_1\sinh\left(\xi\sqrt{-\lambda}\right)+A_2\cosh\left(\xi\sqrt{-\lambda}\right)+\eta/\lambda}\right)\\
&\quad-2\left(\frac{\left(A_1\sin h\left(\xi\sqrt{-\lambda}\right)+A_2\cos h\left(\xi\sqrt{-\lambda}\right)+\eta/\lambda\right)'}{A_1\sin h\left(\xi\sqrt{-\lambda}\right)+A_2\cos h\left(\xi\sqrt{-\lambda}\right)+\eta/\lambda}\right)^2.
\end{aligned}\tag{32}
$$

Then, the exact solution of $(2+1)$-dimensional Ito equation of the form (2) is as follows:

$$
\begin{aligned}
V_{11}&=\frac{1}{6}\left(-13\lambda\pm\sqrt{97\lambda^2-12d}\right)\\
&\quad+\frac{2\eta}{A_1\sinh\left(\xi\sqrt{-\lambda}\right)+A_2\cosh\left(\xi\sqrt{-\lambda}\right)+\eta/\lambda}\Bigg)\\
&\quad+\frac{2\lambda\left(A_1\cos h\left(\xi\sqrt{-\lambda}\right)+A_2\sin h\left(\xi\sqrt{-\lambda}\right)\right)^2}{\left(A_1\sin h\left(\xi\sqrt{-\lambda}\right)+A_2\cos h\left(\xi\sqrt{-\lambda}\right)+\eta/\lambda\right)^2}.
\end{aligned}\tag{33}
$$

By letting $A_1=0$, $\eta=0$, and $A_2>0$ in Equation (33), the particular solution of Equation (2) is as follows:

$$V_{11a} = \frac{1}{6}\left(-13\lambda \pm \sqrt{97\lambda^2 - 12d}\right) + 2\lambda \tan h^2\left(\xi\sqrt{-\lambda}\right). \tag{34}$$

Then, by letting $\lambda = -0.3$, $d = 0.4$, $t = 0$, and $y = 3$, the graph of Equation (34) has the following forms.

From Figure 1, we see the solution of $(2+1)$-dimensional Ito equation with respect to the result 1 and $\lambda = -0.3$, $d = 0.4$, $t = 0$, $y = 3$, $A_1 = 0$, $\eta = 0$, and $A_2 > 0$, and we obtain $y \le 0$, when $-10 \le x \le 10$.

Again letting $A_2 = 0$, $\eta = 0$, and $A_1 > 0$ in Equation (34), we get another particular solution of Equation (2) as follows:

$$V_{11b} = \frac{1}{6}\left(-13\lambda \pm \sqrt{97\lambda^2 - 12d}\right) + 2\lambda \cot h^2\left(\xi\sqrt{-\lambda}\right). \tag{35}$$

Then, by letting $\lambda = -0.3$, $d = 0.4$, $t = 0$, and $y = 3$, the graph of solution of Equation (35) has the following forms.

Figure 2 of the solution (35) concludes the solution of $(2+1)$-dimensional Ito equation of result 1 when $\lambda = -0.3$, $d = 0.4$, $t = 0$, $y = 3$, $A_2 = 0$, $\eta = 0$, $A_1 > 0$, and for $-10 \le x \le 10$, it has no solution in real number on the domain $y > 3$, and the solution is always $y < 3$ in the range of $-10 \le x \le 10$.

Result 2

$$\begin{aligned} c_0 &= \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right), \\ c_1 &= 0, c_2 = -2, b_1 = 0, b_2 = 0, \\ k &= \alpha \pm 2\sqrt{28\lambda^2 - 3d}, \eta = 0. \end{aligned} \tag{36}$$

Now by using the above values of variables and putting into Equation (20), we get the following:

$$V_{1,1}(\xi) = \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) - 2\phi^2. \tag{37}$$

Then, by using Equation (13), the solution of Equation (2) has the form $1/3(-4\lambda \pm \sqrt{28\lambda^2 - 3d}) - 2(G'/G)^2$. Now, when we put the value of G from Equation (5), we get the following:

$$V_{12} = \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) - 2\left(\frac{A_1\sqrt{-\lambda}\cos h\left(\xi\sqrt{-\lambda}\right) + A_2\sqrt{-\lambda}\sin h\left(\xi\sqrt{-\lambda}\right)}{A_1 \sin h\left(\xi\sqrt{-\lambda}\right) + A_2 \cos h\left(\xi\sqrt{-\lambda}\right)}\right)^2. \tag{38}$$

Then, the exact solution of $(2+1)$-dimensional Ito equation of (2), if $\lambda < 0$ with respect to result 2 based Equation (36), is as follows:

$$\frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) - \frac{2\left(A_1\sqrt{-\lambda}\cos h\left(\xi\sqrt{-\lambda}\right) + A_2\sqrt{-\lambda}\sin h\left(\xi\sqrt{-\lambda}\right)\right)^2}{\left(A_1 \sin h\left(\xi\sqrt{-\lambda}\right) + A_2 \cos h\left(\xi\sqrt{-\lambda}\right)\right)^2}. \tag{39}$$

By letting $A_1 = 0$ and $A_2 > 0$ in Equation (39), the particular solution of Equation (2) is as follows:

$$V_{12a} = \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) - \frac{2\left(A_2\sqrt{-\lambda}\sin h\left(\xi\sqrt{-\lambda}\right)\right)^2}{\left(A_2 \cos h\left(\xi\sqrt{-\lambda}\right)\right)^2}. \tag{40}$$

Then, the general solution of $(2+1)$-dimensional Ito equation for hyperbolic function with respect to result on Equation (36), when $A_1 = 0$ and $A_2 > 0$, has the form of

$$= \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) + 2\lambda \tan h^2\left(\xi\sqrt{-\lambda}\right). \tag{41}$$

The graph of the particular solution when $\lambda = -0.3$, $d = 0.4$, $t = 0$, and $y = 3$ is as follows.

From Figure 3, the solution of $(2+1)$-dimensional Ito equation with respect to result 2 and $\lambda = -0.3$, $d = 0.4$, $t = 0$, $y = 3$, $A_1 = 0$, $\eta = 0$, and $A_2 > 0$ implies that condensed form around $x = -3$.

Again letting $A_2 = 0$ and $A_1 > 0$ in Equation (39), we get another particular solution of Equation (2) as follows:

$$V_{12b} = \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) - \frac{2\left(A_1\sqrt{-\lambda}\cos h\left(\xi\sqrt{-\lambda}\right)\right)^2}{\left(A_1 \sin h\left(\xi\sqrt{-\lambda}\right)\right)^2}. \tag{42}$$

Then, the general solution of $(2+1)$-dimensional Ito equation for hyperbolic function with respect to result on Equation (36) when $A_2 = 0$ and $A_1 > 0$ has the form of

$$V_{12b} = \frac{1}{3}\left(-4\lambda \pm \sqrt{28\lambda^2 - 3d}\right) + 2\lambda \cot h^2\left(\xi\sqrt{-\lambda}\right). \tag{43}$$

The graph of the particular solution when $\lambda = -0.3$, $d = 0.4$, $t = 0$, and $y = 0$ as follows.

From Figure 4 above, we see the solution of Equation (43) when $\lambda = -0.3$, $d = 0.4$, $t = 0$, $y = 3$, $A_2 = 0$, and $A_1 > 0$, $-10 \le x \le 10$ has the solution between $-10 \le x \le 10$, and it bends downward.

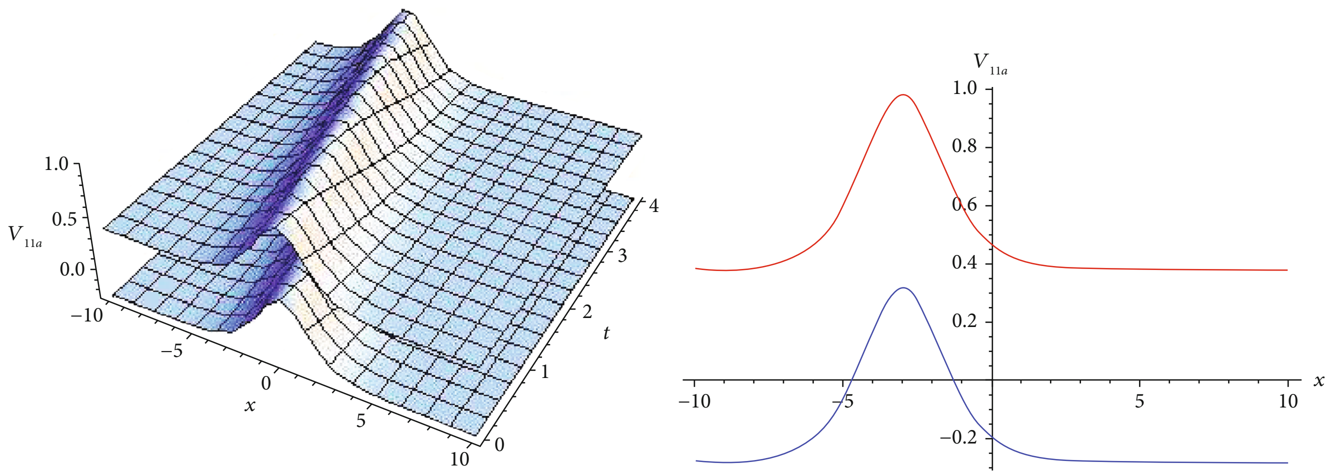

Figure 1: Graphical solution of Equation (34) when $k = \alpha \pm \sqrt{97\lambda^2 - 12d} + \omega$ in 3D and 2D, respectively.

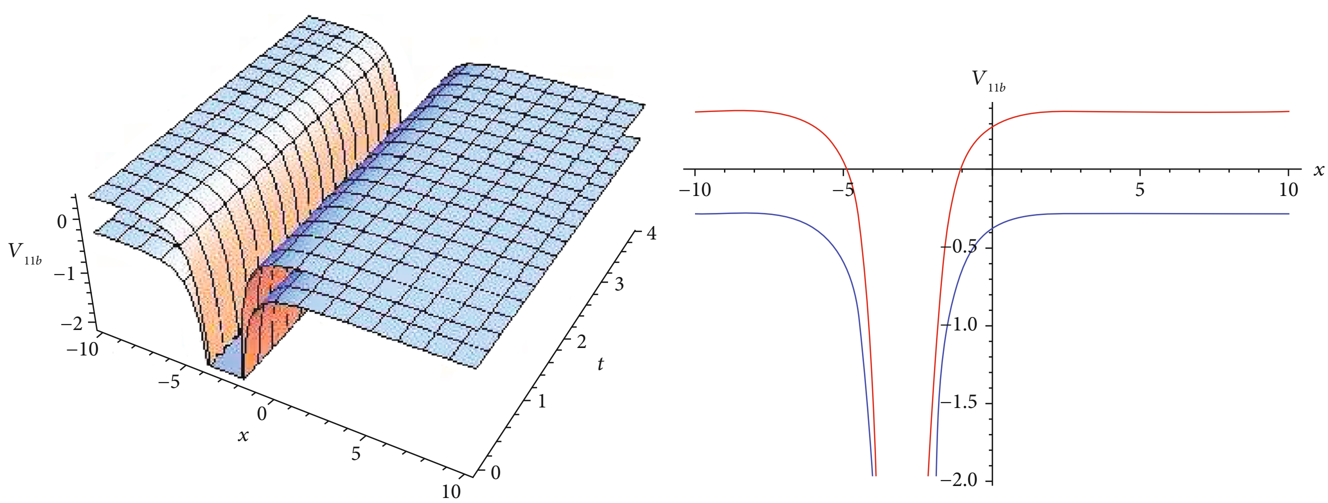

Figure 2: Graphical solution of Equation (35) when $k = \alpha \pm \sqrt{97\lambda^2 - 12d} + \omega$ in 3D and 2D, respectively.

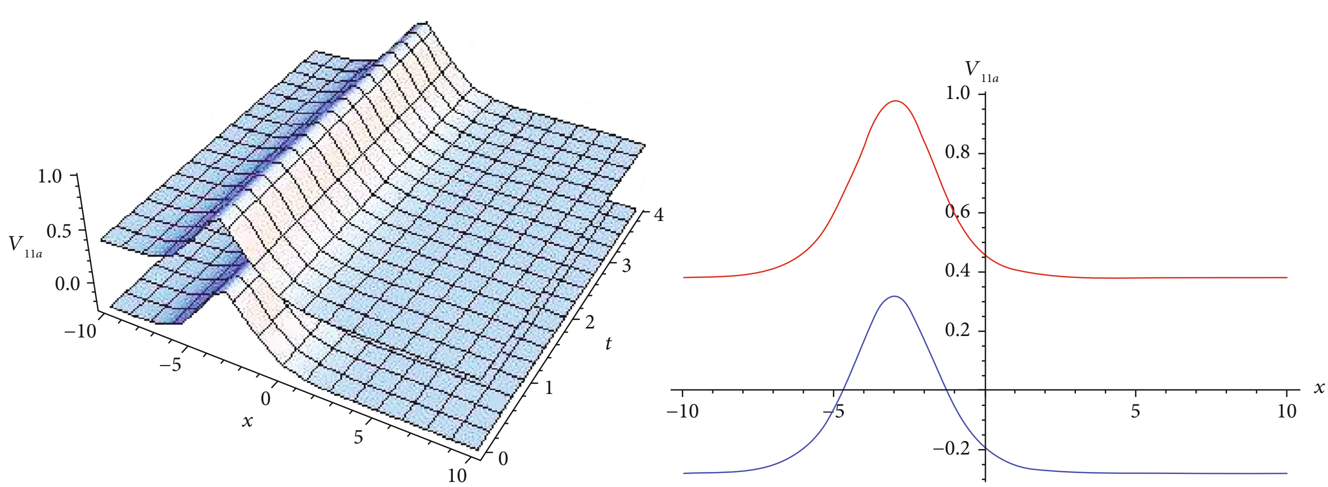

Figure 3: Graphical solution of Equation (41) when $k = \alpha \pm 2\sqrt{28\lambda^2 - 3d}$ in 3D and 2D, respectively.

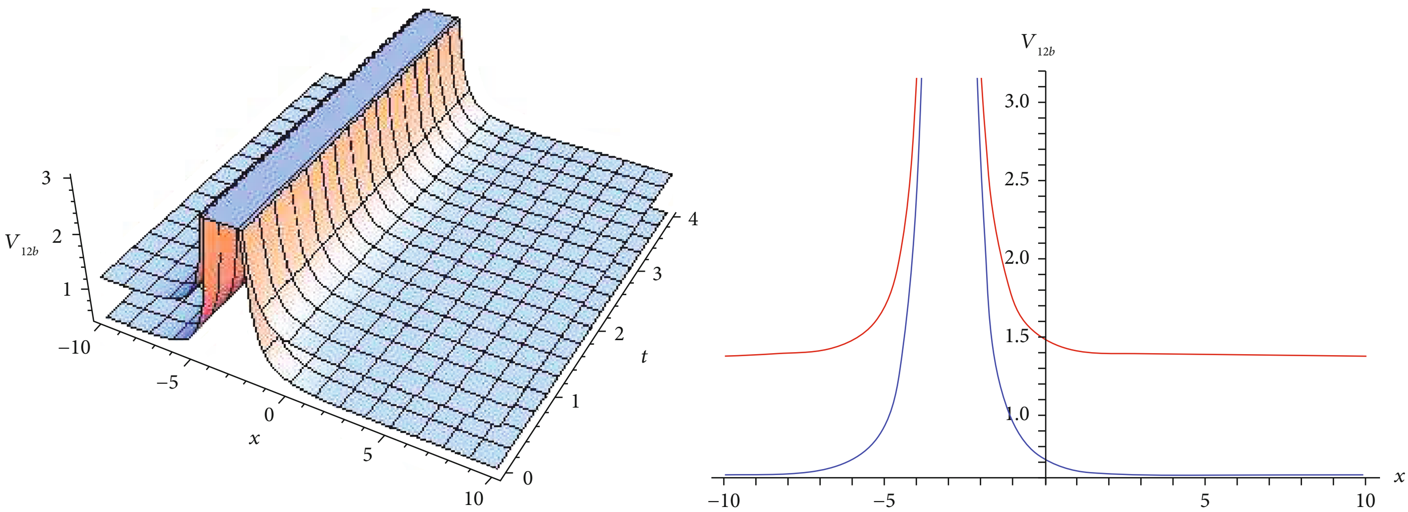

Figure 4: Graphical solution of Equation (43) when $k = \alpha \pm 2\sqrt{28\lambda^2 - 3d}$ in 3D and 2D, respectively.

2.2.2. Case II: If $\lambda > 0$ (Trigonometric Function Solutions). Substituting (6) into (23), we get

$$\begin{aligned} V'' &= 2c_1\phi^3 - 2c_1\eta\beta\phi + 2c_1\lambda\phi - c_1\eta\phi\beta + 2b_1\beta\phi^2 \\ &\quad - \frac{b_1\eta\phi^2\lambda}{\lambda^2\sigma - \eta^2} + \frac{2b_1\eta^2\beta\lambda}{\lambda^2\sigma - \eta^2} - \frac{b_1\eta\lambda^2}{\lambda^2\sigma - \eta^2} \\ &\quad + b_1\beta\lambda + 6c_2\phi^4 - 10c_2\phi^2\eta\beta + 8c_2\phi^2\lambda + \frac{2c_2\eta^2\phi^2\lambda}{\lambda^2\sigma - \eta^2} \\ &\quad - \frac{4c_2\eta^3\beta\lambda}{\lambda^2\sigma - \eta^2} + \frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma - \eta^2} - 4c_2\eta\beta\lambda + 2c_2\lambda^2 \\ &\quad - \frac{2b_2\eta\phi^3\lambda}{\lambda^2\sigma - \eta^2} + \frac{4b_2\phi\eta^2\beta\lambda}{\lambda^2\sigma - \eta^2} - \frac{2b_2\phi\eta\lambda^2}{\lambda^2\sigma - \eta^2} + b_2\lambda\phi\beta. \end{aligned} \tag{44}$$

Using Equation (6), Equation (24) becomes

$$\begin{aligned} V^2 &= \frac{b_1^2\phi^2\lambda}{\lambda^2\sigma - \eta^2} - \frac{2b_1^2\eta\beta\lambda}{\lambda^2\sigma - \eta^2} + \frac{b_1^2\lambda^2}{\lambda^2\sigma - \eta^2} + \frac{2b_1b_2\lambda\phi^3}{\lambda^2\sigma - \eta^2} \\ &\quad - \frac{4b_1b_2\phi\eta\beta\lambda}{\lambda^2\sigma - \eta^2} + \frac{2\phi b_1b_2\lambda^2}{\lambda^2\sigma - \eta^2} + \frac{\phi^4b_2^2\lambda}{\lambda^2\sigma - \eta^2} + \frac{2b_2^2\eta\beta\lambda\phi^2}{\lambda^2\sigma - \eta^2} \\ &\quad + \frac{\phi^2b_2^2\lambda^2}{\lambda^2\sigma - \eta^2} + 2\beta b_1c_0 + 2\beta\phi b_2c_0 + c_0^2 + 2\beta\phi b_1c_1 \\ &\quad + 2\beta\phi^2b_2c_1 + 2\phi c_0c_1 + \phi^2c_1^2 + 2\beta\phi^2b_1c_2 \\ &\quad + 2\beta\phi^3b_2c_2 + 2\phi^2c_0c_2 + 2\phi^3c_1c_2 + \phi^4c_2^2. \end{aligned} \tag{45}$$

Again from Equations (44), (45), and (20), Equation (18) becomes

$$\begin{aligned} &c_o(k - \alpha - \omega) + c_1\phi(k - \alpha - \omega) + b_1\beta(k - \alpha - \omega) \\ &\quad + c_2\phi^2(k - \alpha - \omega) + b_2\phi\beta(k - \alpha - \omega) - 2c_1\phi^3 \\ &\quad - 2c_1\lambda\phi + 3c_1\eta\phi\beta - 2b_1\beta\phi^2 + \frac{b_1\eta\phi^2\lambda}{\lambda^2\sigma - \eta^2} - \frac{2b_1\eta^2\beta\lambda}{\lambda^2\sigma - \eta^2} \\ &\quad + \frac{b_1\eta\lambda^2}{\lambda^2\sigma - \eta^2} - b_1\beta\lambda - 6c_2\phi^4 + 10c_2\phi^2\eta\beta - 6c_2\phi^2\lambda \\ &\quad - \frac{2c_2\eta^2\phi^2\lambda}{\lambda^2\sigma - \eta^2} + \frac{4c_2\eta^3\beta\lambda}{\lambda^2\sigma - \eta^2} - \frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma - \eta^2} + 4c_2\eta\beta\lambda \\ &\quad - 2c_2\lambda\phi^2 - 2c_2\lambda^2 + \frac{2b_2\eta\phi^3\lambda}{\lambda^2\sigma - \eta^2} - \frac{4b_2\phi\beta\eta^2\lambda}{\lambda^2\sigma - \eta^2} + \frac{2b_2\phi\eta\lambda^2}{\lambda^2\sigma - \eta^2} \\ &\quad - b_2\lambda\phi\beta - \frac{3b_1^2\phi^2\lambda}{\lambda^2\sigma - \eta^2} + \frac{6b_1^2\eta\beta\lambda}{\lambda^2\sigma - \eta^2} - \frac{3b_1^2\lambda^2}{\lambda^2\sigma - \eta^2} - \frac{6b_1b_2\lambda\phi^3}{\lambda^2\sigma - \eta^2} \\ &\quad + \frac{12b_1b_2\phi\eta\beta\lambda}{\lambda^2\sigma - \eta^2} - \frac{6\phi b_1b_2\lambda^2}{\lambda^2\sigma - \eta^2} - \frac{3\phi^4b_2^2\lambda}{\lambda^2\sigma - \eta^2} - \frac{6b_2^2\eta\beta\lambda\phi^2}{\lambda^2\sigma - \eta^2} \\ &\quad - \frac{3\phi^2b_2^2\lambda^2}{\lambda^2\sigma - \eta^2} - 6\beta b_1c_0 - 6\beta\phi b_2c_0 - 3c_0^2 - 6\beta\phi b_1c_1 \\ &\quad - 6\beta\phi^2b_2c_1 - 6\phi c_0c_1 - 3\phi^2c_1^2 - 6\beta\phi^2b_1c_2 - 6\beta\phi^3b_2c_2 \\ &\quad - 6\phi^2c_0c_2 - 6\phi^3c_1c_2 - 3\phi^4c_2^2 + d = 0. \end{aligned} \tag{46}$$

Equating the coefficients of the powers of β and ϕ to be zero, we obtain the following system of algebraic equations:

$$\phi^4 : -6c_2 - 3c_2^2 - \frac{3b_2^2\lambda}{\lambda^2\sigma - \eta^2} = 0,$$

$$\phi^3\beta : -6b_2c_2 = 0,$$

$$\phi^3 : -2c_1 - 6c_2c_1 + \frac{2b_2\eta\lambda}{\lambda^2\sigma - \eta^2} - \frac{6b_1b_2\lambda}{\lambda^2\sigma - \eta^2} = 0,$$

$$\phi^2\beta : -2b_1 + 10c_2\eta - 6b_2c_1 - 6c_2b_1 - \frac{6b_2^2\eta\lambda}{\lambda^2\sigma - \eta^2} = 0,$$

$$\phi^2 : c_2(k-\alpha-\omega) - 8c_2\lambda - 3c_1^2 - 6c_0c_2 - \frac{2c_2\eta^2\lambda}{\lambda^2\sigma-\eta^2} - \frac{3b_1^2\lambda}{\lambda^2\sigma-\eta^2} - \frac{3b_2^2\lambda^2}{\lambda^2\sigma-\eta^2} + \frac{b_1\eta\lambda}{\lambda^2\sigma-\eta^2} = 0,$$

$$\phi\beta : b_2(k-\alpha-\omega) + 3c_1\eta - b_2\lambda - 6b_2c_0 - 6c_1b_1 + \frac{4b_2\eta^2\lambda}{\lambda^2\sigma-\eta^2} + \frac{12b_2b_1\eta\lambda}{\lambda^2\sigma-\eta^2} = 0,$$

$$\phi : c_1(k-\alpha-\omega) - 2c_1\lambda - 6c_0c_1 + \frac{2b_2\eta\lambda^2}{\lambda^2\sigma-\eta^2} - \frac{6\phi b_1b_2\lambda^2}{\lambda^2\sigma-\eta^2},$$

$$\beta : b_1(k-\alpha-\omega) - b_1\lambda - 6b_1c_0 + 4c_2\eta\lambda - \frac{2b_1\eta^2\lambda}{\lambda^2\sigma-\eta^2} + \frac{4\eta^3c_2\lambda}{\lambda^2\sigma-\eta^2} - \frac{6\eta\lambda b_1^2}{\lambda^2\sigma+\eta^2} = 0,$$

$$\phi^0 : c_0(k-\alpha-\omega) - 2c_2\lambda^2 - 3c_0^2 + \frac{b_1\eta\lambda^2}{\lambda^2\sigma-\eta^2} - \frac{2c_2\eta^2\lambda^2}{\lambda^2\sigma-\eta^2} - \frac{3b_1^2\lambda^2}{\lambda^2\sigma-\eta^2} + d = 0. \quad (47)$$

Then, by solving the above algebraic equation using Wolfram Mathematica 9.0 software packages, we get the following results.

Result 3

$$c_0 = \frac{1}{6}\left(13\lambda \pm \sqrt{-12d+47\lambda^2}\right), c_1 = 0, c_2 = -2,$$
$$b_1 = \pm\lambda\sqrt{7}\sqrt{\sigma} = 2\eta, b_2 = 0,$$
$$k = \pm\sqrt{-12d+47\lambda^2} + \omega, \eta = \pm\frac{1}{2}\lambda\sqrt{7}\sqrt{\sigma}. \quad (48)$$

Setting the above values of variables into Equation (20), we can get the following form of solution:

$$V(\xi) = \frac{1}{6}\left(13\lambda \pm \sqrt{-12d+47\lambda^2}\right) + 2\eta\beta - 2\phi^2. \quad (49)$$

Then, by using Equation (13), the solution of Equation (2) is a follows:

Then, the exact solution of $(2+1)$-dimensional Ito equation of the form Equation (2) is as follows:

$$V_{21} = \frac{1}{6}\left(13\lambda \pm \sqrt{-12d+47\lambda^2}\right) + \frac{2\eta}{A_1 \sin\left(\xi\sqrt{\lambda}\right) + A_2 \cos\left(\xi\sqrt{\lambda}\right) + \eta/2} + \frac{2\left(A_1\sqrt{\lambda}\cos\left(\xi\sqrt{\lambda}\right) - A_2\sqrt{\lambda}\sin\left(\xi\sqrt{\lambda}\right)\right)^2}{\left(A_1 \sin\left(\xi\sqrt{\lambda}\right) + A_2 \cos\left(\xi\sqrt{\lambda}\right) + \eta/2\right)^2}. \quad (50)$$

By letting $A_1 = 0$, $\eta = 0$, and $A_2 > 0$ in Equation (50), the particular solution of Equation (2) is

$$V_{21a} = \frac{1}{6}\left(13\lambda \pm \sqrt{-12d+47\lambda^2}\right) + 2\lambda \tan^2\left(\xi\sqrt{\lambda}\right). \quad (51)$$

Then, when $\lambda = 0.7$, $d = 0.4$, and $y = 3$, the graph of Equation (51) has the following forms:

Figure 5 shows the exact solution of $(2+1)$-dimensional Ito equation with respect to result 3 when $A_1 = 0$, $\eta = 0$, $A_2 > 0$, $\lambda = 0.7$, $d = 0.4$, $y = 3$, and $-10 \le x \le 10$. This figure implies that the solutions are upward parabolic, and it repeated above x-axis.

Again letting $A_2 = 0$, $\eta = 0$, and $A_1 > 0$ in Equation (50), we get another particular solution of Equation (6) as follows:

$$V_{21b} = \frac{1}{6}\left(13\lambda \pm \sqrt{-12d+47\lambda^2}\right) + 2\lambda \cot^2\left(\xi\sqrt{\lambda}\right). \quad (52)$$

Then, the graph of Equation (52) has the following form:

From the above Figure 6, we see that it is the exact solution of $(2+1)$-dimensional Ito equation with respect to result 3 of Equation (52), when $A_1 > 0$, $\eta = 0$, $A_2 = 0$, $\lambda = 0.7$, $d = 0.4$, $y = 3$, and $-10 \le x \le 10$. And the result of the solution shows it has the form a parabolic upward and crosses the x-axis.

Result 4

$$c_0 = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2 - 3d}\right), c_1 = 0, c_2 = -2,$$
$$b_1 = 0, b_2 = 0, k = \alpha \pm \sqrt{4\lambda^2 - 3d} + \omega, \eta = 0. \quad (53)$$

Then, setting the above values of variables into Equation (20), we can get the following form of solution for Ito equation:

$$V(\xi) = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2 - 3d}\right) - 2\phi^2. \quad (54)$$

Then, by using Equation (13) for ϕ, the solution of Equation (2) is a follows:

$$V(\xi) = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2 - 3d}\right) - 2\left(\frac{G'}{G}\right)^2. \quad (55)$$

But, when we put the value of F of the form Equation (7), we get the following:

$$V(\xi) = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2 - 3d}\right) - 2\left(\frac{\left(A_1 \sin\left(\xi\sqrt{\lambda}\right) + A_2 \cos\left(\xi\sqrt{\lambda}\right)\right)'}{A_1 \sin\left(\xi\sqrt{\lambda}\right) + A_2 \cos\left(\xi\sqrt{\lambda}\right)}\right)^2. \quad (56)$$

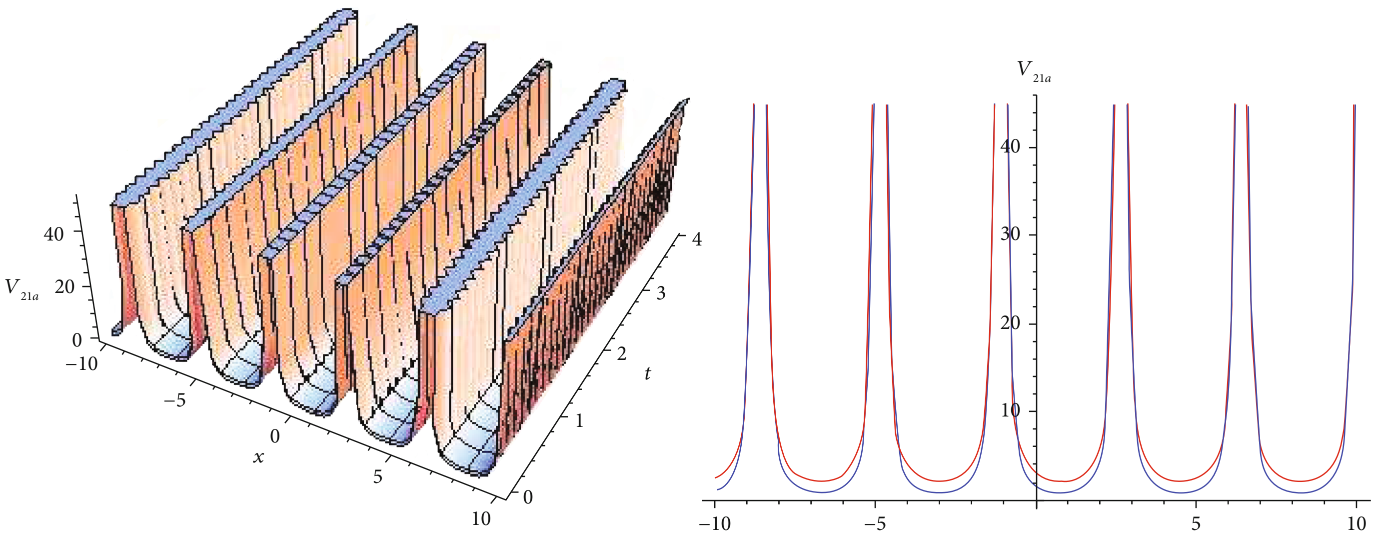

Figure 5: Graphical solution of Equation (51) when $\pm\sqrt{-12d+47\lambda^2}+\omega$ in 3D and 2D, respectively.

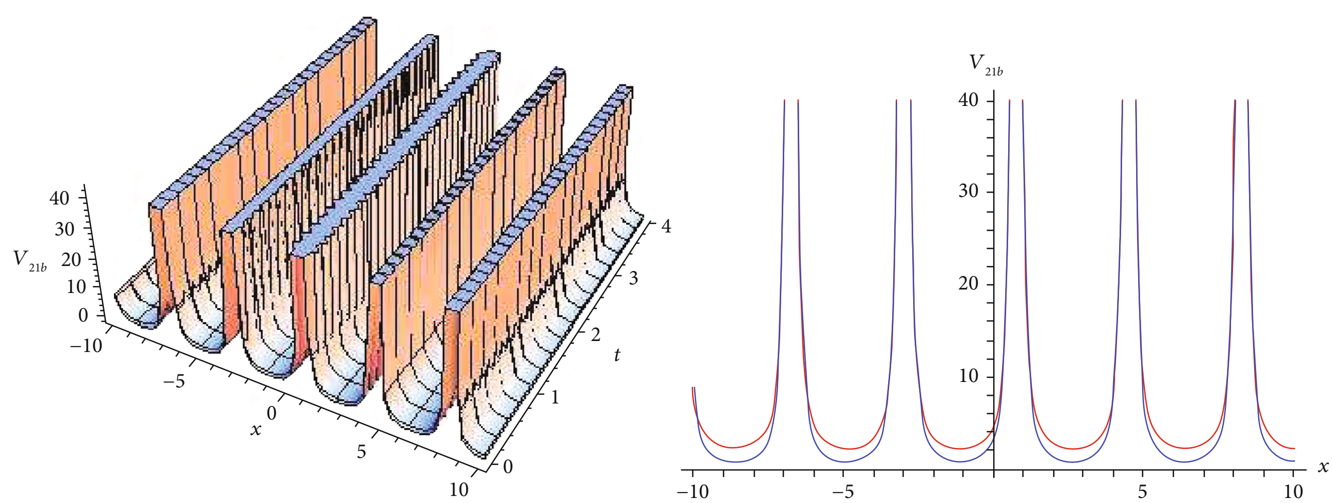

Figure 6: Graphical solution of Equation (52) when $\pm\sqrt{-12d+47\lambda^2}+\omega$ in 3D and 2D, respectively.

Then, finally we get the exact solution of $(2+1)$-dimensional Ito equation with respect to the second case result of the form (2) as follows:

$$V_{22}=\frac{1}{3}\left(-4\lambda\pm\sqrt{4\lambda^2-3d}\right)-2\frac{\left(A_1\sqrt{\lambda}\cos\left(\xi\sqrt{\lambda}\right)-A_2\sqrt{\lambda}\sin\left(\xi\sqrt{\lambda}\right)\right)^2}{\left(A_1\sin\left(\xi\sqrt{\lambda}\right)+A_2\cos\left(\xi\sqrt{\lambda}\right)\right)^2}. \tag{57}$$

By letting $A_1=0$ and $A_2>0$ in Equation (57), the particular solution of Equation (2) is as follows:

$$V_{22a}=\frac{1}{3}\left(-4\lambda\pm\sqrt{4\lambda^2-3d}\right)-2\lambda\tan^2\left(\xi\sqrt{\lambda}\right). \tag{58}$$

Figure 7 shows the exact solution of $(2+1)$-dimensional Ito equation with respect to result 4 when $A_1=0$, $A_2>0$, $\eta=0$, $\lambda=0.7$, $d=0.4$, $y=3$, and at $-10\le x\le 10$, and it makes parabolic bend downward on y-axis.

Again letting $A_2=0$ and $A_1>0$ in Equation (57), we get another particular solution of Equation (2) as follows:

$$V_{22b}=\frac{1}{3}\left(-4\lambda\pm\sqrt{4\lambda^2-3d}\right)-2\frac{\left(A_1\sqrt{\lambda}\cos\left(\xi\sqrt{\lambda}\right)\right)^2}{\left(A_1\sin\left(\xi\sqrt{\lambda}\right)\right)^2},$$
$$V_{22b}=\frac{1}{3}\left(-4\lambda\pm\sqrt{4\lambda^2-3d}\right)-2\lambda\cot^2\left(\xi\sqrt{\lambda}\right). \tag{59}$$

Figure 8 shows the exact solution of $(2+1)$-dimension Ito equation with respect to result 4 when $A_2=0$, $A_1>0$, η

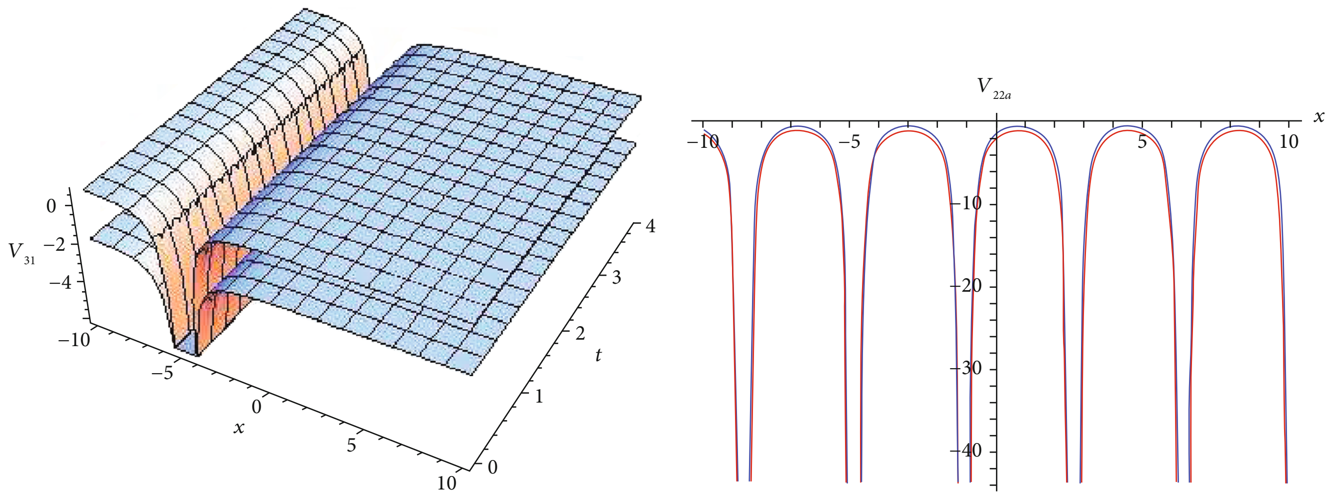

Figure 7: Graphical solution of Equation (58) when $k = \alpha \pm \sqrt{4\lambda^2 - 3d} + \omega$ in 3D and 2D, respectively.

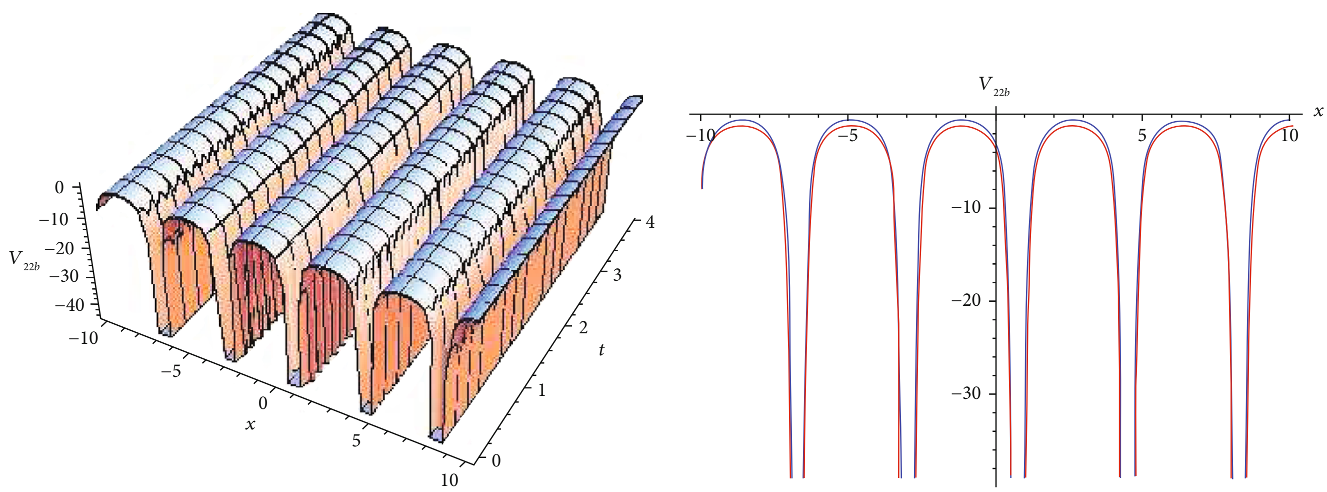

Figure 8: Graphical solution of (44) when $k =$ in 3D and 2D, respectively.

$= 0$, $\lambda = 0.7$, $d = 0.4$, $y = 3$, and at $-10 \le x \le 10$, and it lies always below x-axis in the form of parabolic.

2.2.3. Case III: When $\lambda = 0$ (Rational Function Solutions). Substituting Equation (6) into Equation (23), we get

$$\begin{aligned} V'' = {} & 2c_1\phi^3 - 3c_1\eta\beta\phi + 2c_1\lambda\phi + 2b_1\beta\phi^2 - \frac{b_1\phi^2}{A_1^2 - 2\eta A_2} \\ & + \frac{2b_1\eta\beta}{A_1^2 - 2\eta A_2} + b_1\beta\lambda + 6c_2\phi^4 - 10c_2\phi^2\eta\beta + 8c_2\phi^2\lambda \\ & + \frac{2c_2\eta^2\phi^2}{A_1^2 - 2\eta A_2} - \frac{4c_2\eta^3\beta}{A_1^2 - 2\eta A_2} - 4c_2\eta\beta\lambda + 2c_2\lambda^2 \\ & - \frac{2b_2\eta\phi^3}{A_1^2 - 2\eta A_2} + \frac{4b_2\phi\eta^2\beta}{A_1^2 - 2\eta A_2} + b_2\lambda\phi\beta. \end{aligned} \tag{60}$$

Again also substituting Equation (6) into Equation (24), we get

$$\begin{aligned} V^2 = {} & b_1^2\left(\frac{\phi^2 - 2\eta\beta}{A_1^2 - 2\eta A_2}\right) + 2\phi b_1 b_2\left(\frac{\phi^2 - 2\eta\beta}{A_1^2 - 2\eta A_2}\right) \\ & + \phi^2 b_2^2\left(\frac{\phi^2 - 2\eta\beta}{A_1^2 - 2\eta A_2}\right) + 2\beta b_1 c_0 + 2\beta\phi b_2 c_0 \\ & + c_0^2 + 2\beta\phi b_1 c_1 + 2\beta\phi^2 b_2 c_1 + 2\phi c_1 c_0 + \phi^2 c_1^2 \\ & + 2\beta\phi^2 b_1 c_2 + 2\beta\phi^3 b_2 c_2 + 2\phi^2 c_0 c_2 + 2\phi^3 c_1 c_2 + \phi^4 c_2^2 \end{aligned} \tag{61}$$

$$\begin{aligned} = {} & \frac{b_1^2\phi^2}{A_1^2 - 2\eta A_2} - \frac{2b_1^2\eta\beta}{A_1^2 - 2\eta A_2} + \frac{2b_1 b_2\phi^3}{A_1^2 - 2\eta A_2} - \frac{4\phi b_1 b_2\eta\beta}{A_1^2 - 2\eta A_2} \\ & + \frac{\phi^4 b_2^2}{A_1^2 - 2\eta A_2} - \frac{2\phi^2 b_2^2\eta\beta}{A_1^2 - 2\eta A_2} + 2\beta b_1 c_0 + 2\beta\phi b_2 c_0 \\ & + c_0^2 + 2\beta\phi b_1 c_1 + 2\beta\phi^2 b_2 c_1 + 2\phi c_1 c_0 + \phi^2 c_1^2 + 2\beta\phi^2 b_1 c_2 \\ & + 2\beta\phi^3 b_2 c_2 + 2\phi^2 c_0 c_2 + 2\phi^3 c_1 c_2 + \phi^4 c_2^2. \end{aligned} \tag{62}$$

From Equation (20), (60), and (62), Equation (18) becomes

$$\begin{aligned} V(\xi) &= c_o(k-\alpha-\omega) + c_1\phi(k-\alpha-\omega) + b_1\beta(k-\alpha-\omega) \\ &+ c_2\phi^2(k-\alpha-\omega) + b_2\phi\beta(k-\alpha-\omega) - 2c_1\phi^3 \\ &+ 3c_1\eta\beta\phi - 2c_1\lambda\phi - 2b_1\beta\phi^2 + \frac{b_1\eta\phi^2}{A_1^2-2\eta A_2} \\ &- \frac{2b_1\eta\beta}{A_1^2-2\eta A_2} - b_1\beta\lambda - 6c_2\phi^4 + 10c_2\phi^2\eta\beta - 8c_2\phi^2\lambda \\ &- \frac{2c_2\eta^2\phi^2}{A_1^2-2\eta A_2} + \frac{4c_2\eta^3\beta}{A_1^2-2\eta A_2} + 4c_2\eta\beta\lambda - 2c_2\lambda^2 \\ &+ \frac{2b_2\eta\phi^3}{A_1^2-2\eta A_2} - \frac{4b_2\phi\eta^2\beta}{A_1^2-2\eta A_2} - b_2\lambda\phi\beta - \frac{3b_1^2\phi^2}{A_1^2-2\eta A_2} \\ &+ \frac{6b_1^2\eta\beta}{A_1^2-2\eta A_2} - \frac{6b_1b_2\phi^3}{A_1^2-2\eta A_2} + \frac{12\phi b_1b_2\eta\beta}{A_1^2-2\eta A_2} \\ &- \frac{3\phi^4b_2^2}{A_1^2-2\eta A_2} + \frac{6\phi^2b_2^2\eta\beta}{A_1^2-2\eta A_2} - 6\beta b_1c_0 - 6\beta\phi b_2c_0 \\ &- 3c_0^2 - 6\beta\phi b_1c_1 - 6\beta\phi^2b_2c_1 - 6\phi c_1c_0 - 3\phi^2c_1^2 \\ &- 6\beta\phi^2b_1c_2 - 6\beta\phi^3b_2c_2 - 6\phi^2c_0c_2 - 6\phi^3c_1c_2 - 3\phi^4c_2^2. \end{aligned} \tag{63}$$

Then, by setting the coefficients of the polynomial to be zero, we obtain the following sets of algebraic.

$$\begin{aligned} \phi^4 &: -6c_2 - 3c_2^2 - \frac{3b_2^2}{A_1^2-2\eta A_2} = 0, \\ \phi^3\beta &: -6b_2c_2 = 0, \\ \phi^3 &: -2c_1 - 6c_2c_1 + \frac{2b_2\eta}{A_1^2-2\eta A_2} - \frac{6b_1b_2}{\lambda^2\sigma-\eta^2} = 0, \\ \phi^2\beta &: -2b_1 + 10c_2\eta - 6b_2c_1 - 6c_2b_1 + \frac{6b_2^2\eta}{A_1^2-2\eta A_2} = 0, \\ \phi^2 &: c_2(k-\alpha-\omega) - 8c_2\lambda - 3c_1^2 - 6c_0c_2 \\ &\quad - \frac{2c_2\eta^2}{A_1^2-2\eta A_2} - \frac{3b_1^2}{A_1^2-2\eta A_2} + \frac{b_1\eta}{A_1^2-2\eta A_2} = 0, \\ \phi\beta &: b_2(k-\alpha-\omega) + 3c_1\eta - b_2\lambda - 6b_2c_0 - 6c_1b_1 \\ &\quad - \frac{4b_2\eta^2}{A_1^2-2\eta A_2} + \frac{12b_2b_1\eta}{A_1^2-2\eta A_2} = 0, \\ \phi &: c_1(k-\alpha-\omega) - 2c_1\lambda - 6c_0c_1 = 0, \\ \beta &: b_1(k-\alpha-\omega) - b_1\lambda - 6b_1c_0 + 4c_2\eta\lambda - \frac{2b_1\eta}{A_1^2-2\eta A_2} \\ &\quad + \frac{4\eta^3c_2}{A_1^2-2\eta A_2} + \frac{6\eta b_1^2}{A_1^2-2\eta A_2}, \\ \phi^0 &: c_0(k-\alpha-\omega) - 2c_2\lambda^2 - 3c_0^2 + d = 0. \end{aligned} \tag{64}$$

Then, by solving previous algebraic equation using Wolfram Mathematica 9.0 software packages, we get the following results.

Result 5

$$\begin{aligned} c_0 &= \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2-3d}\right), c_1 = 0, c_2 = -2, b_1 = 0, \\ b_2 &= 0, k = \alpha \pm 2\sqrt{4\lambda^2-3d} + \omega, \eta = 0. \end{aligned} \tag{65}$$

Then, setting Equation (65) values of variables into Equation (20), we can get the following form of solution for Ito equation:

$$V_{31} = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2-3d}\right) - 2\left(\frac{\left(\eta/2\xi^2 + A_1\xi + A_2\right)'}{\eta/2\xi^2 + A_1\xi + A_2}\right)^2. \tag{66}$$

When the value of $\eta = 0$, then the solution of Equation (66) is as follows:

$$V_{31} = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2-3d}\right) - 2\left(\frac{A_1}{A_1\xi + A_2}\right)^2. \tag{67}$$

Finally, we get the exact solution of $(2+1)$-dimensional space time equation with respect to the first result of case 3 in Equation (65) as follows:

$$V_{31} = \frac{1}{3}\left(-4\lambda \pm \sqrt{4\lambda^2-3d}\right) - \frac{2A_1^2}{(A_1\xi + A_2)^2}. \tag{68}$$

When $\lambda = 0$, we have

$$V_{31} = \pm\sqrt{-3d} - \frac{2A_1^2}{(A_1\xi + A_2)^2}. \tag{69}$$

Using the parameters $d = -0.5$, $A_1 = 1$, $A_2 = 2$, and $t = 0$, the graph of Equation (69) is as follows:

Figure 9 represents the exact solution of $(2+1)$-dimensional space time with respect to the parameters $d = -0.5$, $A_1 = 1$, $A_2 = 2$, $t = 0$, and $y = 3$ which has maximum limit on the y-axis.

Result 6

$$\begin{aligned} c_0 &= \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2-12d}\right), c_1 = 0, c_2 = -2, \\ b_1 &= 2\left(\lambda A_2 \pm \sqrt{\lambda\left(\lambda A_2^2 - A_1^2\right)}\right) = 2\eta, \end{aligned} \tag{70}$$

$$\begin{aligned} b_2 &= 0, k = \frac{\alpha A_1^2 \pm \sqrt{73\lambda^2-12d}A_1^2 + \omega A_1^2}{A_1^2}, \\ \eta &= \lambda A_2 \pm \sqrt{\lambda\left(\lambda A_2^2 - A_1^2\right)}. \end{aligned} \tag{71}$$

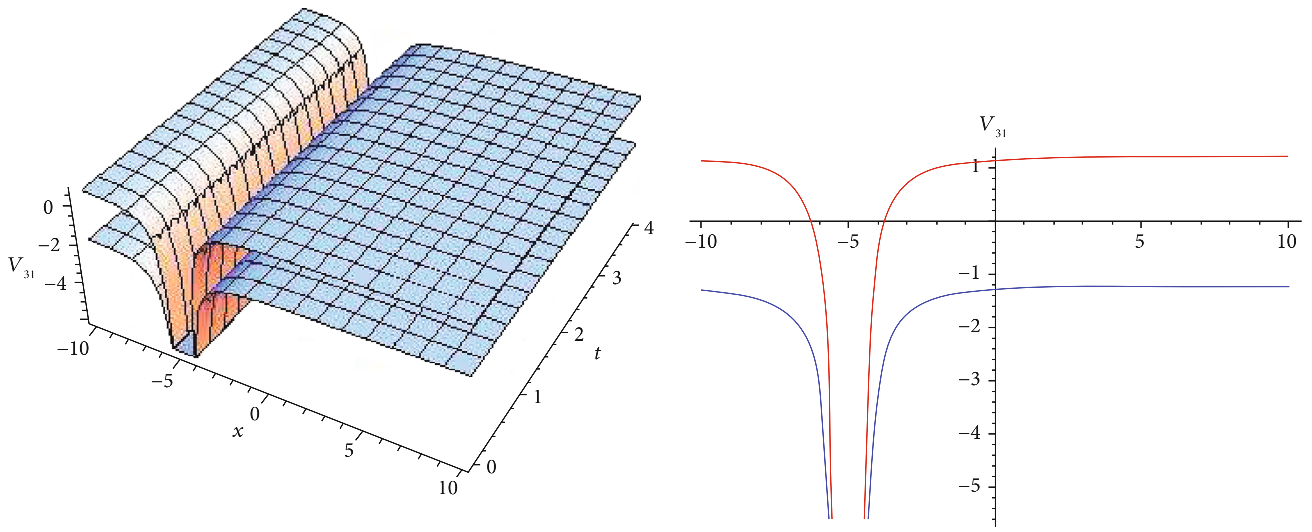

FIGURE 9: Graphical solution of (69) when $k = \alpha \pm 2\sqrt{4\lambda^2 - 3d} + \omega$ in 3D and 2D, respectively.

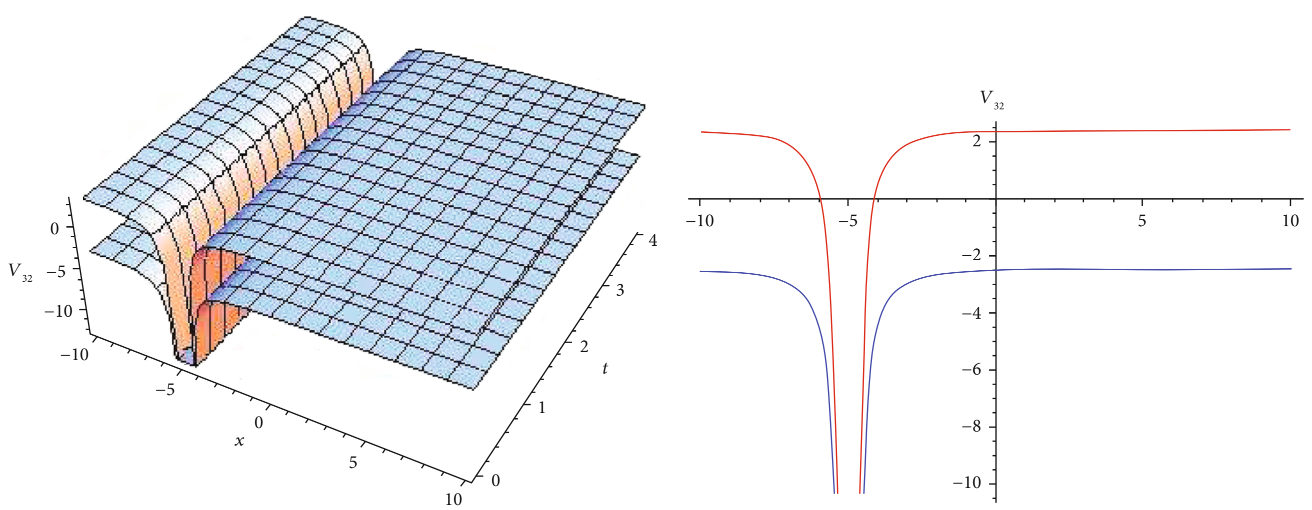

FIGURE 10: Graphical solution of (77) when $k = \alpha A_1^2 \pm \sqrt{73\lambda^2 - 12d}A_1^2 + \omega A_1^2/A_1^2$ in 3D and 2D, respectively.

Then, setting the Equation (70) values of variables into Equation (20), we can get the following form of solution for Ito equation:

$$V_{32} = \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2 - 12d}\right) + 2\eta\beta - 2\phi^2. \qquad (72)$$

Then by using Equation (13) for ϕ and β, the simplification of Equation (72) leads to

$$V_{32} = \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2 - 12d}\right) + 2\eta\frac{1}{G} - 2\left(\frac{G'}{G}\right)^2. \qquad (73)$$

When we put the value of G from Equation (8) in Equation (73),

$$V_{32} = \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2 - 12d}\right) + \frac{2\eta}{\eta/2\xi^2 + A_1\xi + A_2} - 2\left(\frac{\eta\xi + A_1}{\eta/2\xi^2 + A_1\xi + A_2}\right)^2. \qquad (74)$$

Hence, we get the exact solution of $(2+1)$-dimensional space time with respect to result of (70) as follows:

$$V_{32} = \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2 - 12d}\right) + \frac{2\eta}{\eta/2\xi^2 + A_1\xi + A_2} - \frac{2(\eta\xi + A_1)^2}{\left(\eta/2\xi^2 + A_1\xi + A_2\right)^2}. \qquad (75)$$

By letting $\eta = 0$, $A_1 > 0$, and $A_2 > 0$ in Equation (75), the particular solution of Equation (2) with respect to (70) is as follows:

$$V_{32a} = \frac{1}{6}\left(-11\lambda \pm \sqrt{73\lambda^2 - 12d}\right) - \frac{2A_1^2}{(A_1\xi + A_2)^2}. \quad (76)$$

When $\lambda = 0$, we get

$$V_{32a} = \pm\sqrt{-12d}\Big) - \frac{2A_1^2}{(A_1\xi + A_2)^2}. \quad (77)$$

Then, by setting $d = -0.5$, $A_1 = 1$, $A_2 = 2$, and $t = 0$, the graph of (77) is a follows:

Figure 10 reveals the exact solution of $(2+1)$-dimensional space time with respect to result 6 when the parameters $d = -0.5$, $A_1 = 1$, $A_2 = 2$, and $t = 0$ and in similar way as figure 9 fixes its maximum from below on y-axis.

3. Conclusion

In this article, the solution of NLPDE of $(2+1)$-dimensional space time is analytically solved by special expansion method. The special expansion method is one of the direct method to solve the nonlinear partial differential equations. To apply the method, we used the combined form $(G'/G, 1/G)$ expansion method. The method has a great advantage to solve the NLPDE which leads us to get exact solutions for three different cases of λ. The special expansion method was applied on the proposed equations and leads to find for hyperbolic functions, trigonometric functions, and rational functions. Furthermore, the obtained results are exactly fit exact solutions which solve the complicity of finding the solutions for NLPDE. The obtained solution for each results was illustrated by graphical plots using the Wolfram Mathematica 9.0 software, and also, the surface plane models are constructed side by side to show the physical and geometrical interpretations on the ground. Finally, the method is powerful and effective to solve NLPDE.

Data Availability

All required data were included in the manuscript and cited appropriately when it was required.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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