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Application of Quick Simplex Method on the Dual Simplex Method (A New Approach)

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Abstract

In this article, we suggest a new approach while solving Dual simplex method using Quick Simplex Method. Quick Simplex Method attempts to replace more than one basic variable simultaneously so it involves less iteration or at the most equal number than in the standard Dual Simplex Method.
This has been illustrated by giving the solution of solving Dual Simplex Method Problems.
It is also shown that either the iterations required are the same or less but iterations required are never more than those of the Dual Simplex Method.

Keywords: Basic solution; optimum solution; feasible solution; dual simplex method; key determinant; constraints; net evaluation.

1 Introduction

There are various methods to obtain the solution of the linear programming problem such as; i) Graphical Method ii) Simplex Method iii) Dual Simplex Method [6].

Dual Simplex Method: In Simplex Method, we have already seen that every basic solution with all Z_j-$C_j \geq 0$ will not be feasible, but any feasible solution with all Z_j-$C_j \geq 0$ will certainly be an optimal solution. Such

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type of typical problems, for which it is possible to find infeasible but better than optimal initial basic solution (with all Z_j-$C_j \geq 0$), can be solved more easily by Dual Simplex Method. Such a situation is recognized by first expressing the constraints in the form (≤) and the objective function in the maximization form. After adding the slack variables and putting the problem in the table form, if any of the right hand side elements are negative and if the optimality condition is satisfied, then the problem can be solved by the Dual Simplex Method. It is important to note that by this arrangement, negative element on the right hand side signifies that the corresponding slack variable is negative. This means that the problem starts with optimal but infeasible basic solution as required by Dual Simplex Method. In this method, we shall proceed towards feasibility maintaining optimality and at the iteration where the basic solution becomes feasible, it becomes the optimal basic feasible solution also.

The Dual Simplex Method is very similar to the regular simplex method. In fact, once they are started, the only difference lies in the criterion used for selecting a vector to enter the basis and to leave the basis. Also, it is to be noted that in the Dual Simplex Method, we first determine the vector to leave the basis and then the vector to enter the basis. This is just reverse of what is done in the simplex method. The Dual Simplex Method yields an optimal solution to the given linear programming problem in a finite number of steps, provided no basis had to be repeated.

Since the Dual Simplex Method deals with the primal problem as if the simplex method was being applied simultaneously to its dual problem, and the criteria used for inserting and leaving vectors are those for the dual rather than the primal problem; that is why this method is called the Dual Simplex Method.

2 Quick Simplex Method [1-10]

Quick Simplex Method attempts to replace more than one basic variable simultaneously. Here we are applying this method on Dual Simplex Method.

3 Statement of the Problem –I

Use dual simplex method to solve the following linear programming problem:

$$\text{Min } Z = 80\, x_1 + 60\, x_2 + 80\, x_3$$

Subject to

$$x_1 + 2x_2 + 3\, x_3 \geq 4$$
$$2x_1 + 3\, x_3 \geq 3$$
$$2x_1 + 2x_2 + x_3 \geq 4$$
$$4x_1 + x_2 + x_3 \geq 6$$
$$x_j \geq 0$$

4 Solution of the Problem by Dual Simplex Method [6]

$$\text{Max } Z = -80\, x_1 - 60\, x_2 - 80\, x_3 + 0\, x_4 + 0\, x_5 + 0x_6 + 0x_7$$

Subject to

$$-x_1 - 2x_2 - 3\, x_3 + x_4 = -4$$
$$-2x_1 - 3\, x_3 + x_5 = -3$$
$$-2x_1 - 2x_2 - x_3 + x_6 = -4$$
$$-4x_1 - x_2 - x_3 + x_7 = -6$$
$$x_j \geq 0$$

Step I Initial Table

C_B	Y_B	X_B	-80	-60	-80	0	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_4	-4	-1	**-2**	-3	1	0	0	0
0	y_5	-3	-2	0	-3	0	1	0	0
0	y_6	-4	-2	-2	**-1**	0	0	1	0
0	y_7	**-6**	**-4**	-1	-1	0	0	0	1
	Z_j-C_j		80	60	80	0	0	0	0
	$(Z_j$-$C_j)/y_{kj}$		**-20**	-60	-80				

Step II First Iteration Entering y_1 and Outgoing vector is y_7

C_B	Y_B	X_B	-80	-60	-80	0	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_4	-5/2	0	-7/4	-11/4	1	0	0	-1/4
0	y_5	0	0	1/2	-5/2	0	1	0	-1/2
0	y_6	-1	0	-3/2	-1/2	0	0	1	-1/2
-80	y_1	3/2	1	1/4	1/4	0	0	0	-1/4
	Z_j-C_j		0	40	60	0	0	0	20
	$(Z_j$-$C_j)/y_{kj}$			-160/7	**-240/11**				-40

Step III Second Iteration Entering y_3 and Outgoing vector is y_4

C_B	Y_B	X_B	-80	-60	-80	0	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
-80	y_3	10/11	0	7/11	1	-4/11	0	0	1/11
0	y_5	25/11	0	23/11	0	-10/11	1	0	-3/11
0	y_6	**-6/11**	0	**-13/11**	0	-2/11	0	1	-5/11
-80	y_1	14/11	**1**	1/11	0	1/11	0	0	-3/11
	Z_j-C_j		0	20/11	0	240/11	0	0	100/11
	$(Z_j$-$C_j)/y_{kj}$			**-20/13**		-120			-200/13

Step IV Third Iteration Entering y_2 and Outgoing vector is y_6

C_B	Y_B	X_B	-80	-60	-80	0	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
-80	y_3	8/13	0	0	1	-6/13	0	7/13	-2/13
0	y_5	17/13	0	0	0	-16/13	1	23/13	-14/13
-60	y_2	6/13	0	1	0	2/13	0	-11/13	5/13
-80	y_1	16/13	1	0	0	1/13	0	1/13	-4/13
	Z_j-C_j		0	0	0	280/13	0	20/13	180/13

Using Dual Simplex Method we get the basic solution x_1=16/13, x_2=6/13, x_3=8/13 and MinZ= 2280/13 which is feasible so this is the required optimal solution.

5 Here We Apply Quick Simplex Method on Dual Simplex Method

Step I Initial Table

C_B	Y_B	X_B	**-80**	**-60**	**-80**	**0**	**0**	**0**	**0**
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_4	-4	-1	**-2**	-3	1	0	0	0
0	y_5	-3	-2	0	-3	0	1	0	0
0	y_6	-4	-2	-2	**-1**	0	0	1	0
0	y_7	**-6**	**-4**	-1	-1	0	0	0	1
	Z_j-C_j		80	60	80	0	0	0	0
	$(Z_j$-$C_j)/y_{kj}$		**-20**	**-60**	**-80**				

Here we introduce y_1, y_2, y_3 simultaneously and outgoing vectors are y_7, y_4, y_6.

To find new values in X_B column, we use the formulae from Quick Simplex Method. [1-4,7].

$$R=\begin{vmatrix}-1 & -2 & -3\\ -2 & -2 & -1\\ -4 & -1 & -1\end{vmatrix} = 13 \qquad d_1^{***} = \begin{vmatrix}-4 & -2 & -3\\ -4 & -2 & -1\\ -6 & -1 & -1\end{vmatrix} / R = \frac{16}{13}$$

$$d_2^{***} = \begin{vmatrix}-1 & -4 & -3\\ -2 & -4 & -1\\ -4 & -6 & -1\end{vmatrix} / R = \frac{6}{13} \qquad d_3^{***} = \begin{vmatrix}-1 & -2 & -4\\ -2 & -2 & -4\\ -4 & -1 & -6\end{vmatrix} / R = \frac{8}{13}$$

$$d_4^{***} = \begin{vmatrix}-1 & -2 & -3 & -4\\ -2 & -2 & -1 & -4\\ -4 & -1 & -1 & -6\\ -2 & 0 & -3 & -3\end{vmatrix} / R = \frac{17}{13}$$

Column $x_B = \begin{vmatrix}y_4\\ y_5\\ y_6\\ y_7\end{vmatrix} = \begin{vmatrix}-4\\ -3\\ -4\\ -6\end{vmatrix}$ can be replace by new $x_B = \begin{vmatrix}y_3\\ y_5\\ y_2\\ y_1\end{vmatrix} = \begin{vmatrix}\frac{8}{13}\\ \frac{17}{13}\\ \frac{6}{13}\\ \frac{16}{13}\end{vmatrix}$

Since we get all the entries of X_B column positive so we can get the basic feasible solution. In this way we can find new values for each vector using Quick Simplex Method.

Step II Final Table

C_B	Y_B	X_B	**-80**	**-60**	**-80**	**0**	**0**	**0**	**0**
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
-80	y_3	8/13	0	0	1	-6/13	0	7/13	-2/13
0	y_5	17/13	0	0	0	-16/13	1	23/13	-14/13
-60	y_2	6/13	0	1	0	2/13	0	-11/13	5/13
-80	y_1	16/13	1	0	0	1/13	0	1/13	-4/13
	Z_j-C_j		0	0	0	280/13	0	20/13	180/13

Since the basic solution x_1=16/13, x_2=6/13,x_3=8/13 and MinZ= 2280/13 is feasible at this stage, so this is the required optimal solution.

6 Statement of the Problem –II

Use dual simplex method to solve the following linear programming problem:

$$\text{Min } Z = 3x_1 + 2x_2 + x_3 + 4x_4$$

$$2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$$
$$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$$
$$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$$
$$x_j \geq 0$$

VII Solution of the problem by Dual Simplex Method:

$$\text{Max } Z = -3x_1 - 2x_2 - x_3 - 4x_4 + 0x_5 + 0x_6 + 0x_7$$

$$-2x_1 - 4x_2 - 5x_3 - x_4 + x_5 = -10$$
$$-3x_1 + x_2 - 7x_3 + 2x_4 + x_6 = -2$$
$$-5x_1 - 2x_2 - x_3 - 6x_4 + x_7 = -15$$
$$x_j \geq 0$$

Using Dual Simplex Method we get the basic solution $x_1 = 65/23$, $x_2=0$, $x_3=20/23$, $x_4=0$ and Min Z= 215/23 which is feasible so this is the required optimal solution.

VIII here we apply quick simplex method on Dual Simplex Method.

Step I Initial Table

C_B	Y_B	X_B	**-3**	**-2**	**-1**	**-4**	**0**	**0**	**0**
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_5	-10	-2	-4	**-5**	-1	1	0	0
0	y_6	-2	-3	1	-7	2	0	1	0
0	y_7	-15	**-5**	-2	-1	-6	0	0	1
	Z_j-C_j		3	2	1	4	0	0	0
	$(Z_j-C_j)/y_{kj}$		**-3/5**	-1	**-1**	-2/3	**0**	**0**	**0**

Here we introduce y_1, y_2, y_3 simultaneously and outgoing vectors are y_7, y_5, y_6 but we get x_B value negative so we introduce only two vectors y_1, y_3 simultaneously and outgoing vectors are y_7, y_5.

To find new values in X_B column,we use the formulae from Quick Simplex Method, [2,3]

$$R = \begin{vmatrix} -2 & -5 \\ -5 & -1 \end{vmatrix} = -23\ , \qquad C_1^{**} = \begin{vmatrix} -10 & -5 \\ -15 & -1 \end{vmatrix} / R = 65/23$$

$$C_2^{**} = \begin{vmatrix} -2 & -10 \\ -5 & -15 \end{vmatrix} / R = 20/23\ , \qquad C_3^{**} = \frac{\begin{vmatrix} -2 & -5 & -10 \\ -5 & -1 & -15 \\ -3 & -7 & -2 \end{vmatrix}}{R} = 289/23$$

Since we get all the entries of X_B column positive so we can get the basic feasible solution. In this way we can find new values for each vector using Quick Simplex Method.

Step II Final Table

C_B	Y_B	X_B	-3	-2	-1	-4	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
-1	y_3	20/23	0	16/23	1	-7/23	-5/23	0	2/23
0	y_6	289/23	0	153/23	0	84/23	-32/23	1	-1/23
-3	y_1	65/23	1	6/23	0	29/23	1/23	0	-5/23
	Z_j-C_j								

Since the basic solution $x_1 = 65/23$, $x_2=0$, $x_3=20/23$, $x_4=0$ and Min Z= 215/23 which is feasible so this is the required optimal solution.

IX Statement of the problem –III

Use dual simplex method to solve the following linear programming problem:

Min $Z = x_1 + 2x_2 + 3x_3$
Subject to
$2x_1 - x_2 + x_3 \geq 4$
$x_1 + x_2 + 2x_3 \geq 8$
$x_2 - x_3 \geq 2$
$x_j \geq 0$

X Solution of the problem by Dual Simplex Method:

Min $Z = -x_1 - 2x_2 - 3x_3 + 0x_4 + 0x_5 + 0x_6$
Subject to
$-2x_1 + x_2 - x_3 + x_4 = -4$
$-x_1 - x_2 - 2x_3 + x_5 = -8$
$-x_2 + x_3 + x_6 = -2$
$x_j \geq 0$

Using Dual Simplex Method we get the basic solution $x_1 = 6$, $x_2 = 2$, $x_3 = 0$ and Min Z =10 which is feasible so this is the required optimal solution.

XI Here we Apply Quick Simplex Method on Dual Simplex Method.

Step I Initial Table

			-1	-2	-3	0	0	0
C_B	Y_B	X_B	y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	-4	-2	1	-1	1	0	0
0	y_5	-8	-1	-1	-2	0	1	0
0	y_6	-2	0	**-1**	1	0	0	1
	Z_j-C_j		1	2	3	0	0	0
	$(Z_j$-$C_j) / y_{kj}$		**-1**	**-2**	**-3/2**			

Here we introduce y_1, y_2 simultaneously and outgoing vectors are y_5, y_6

To find new values in X_B column, we use the formulae from Quick Simplex Method, [2-4].

$$R = \begin{vmatrix} -1 & -1 \\ 0 & -1 \end{vmatrix} = 1 \quad C_1^{**} = \begin{vmatrix} -8 & -1 \\ -2 & -1 \end{vmatrix} / R = 6$$

$$C_2^{**} = \begin{vmatrix} -1 & -8 \\ 0 & -2 \end{vmatrix} / R = 2\,, \qquad C_3^{**} = \frac{\begin{vmatrix} -1 & -1 & -8 \\ 0 & -1 & -2 \\ -2 & 1 & -4 \end{vmatrix}}{R} = 6$$

Since we get all the entries of X_B column positive so we can get the basic feasible solution. In this way we can find new values for each vector using Quick Simplex Method.

Step II Final Table

C_B	Y_B	X_B	**-1**	**-2**	**-3**	**0**	**0**	**0**
			y_1	y_2	y_3	y_4	y_5	y_6
0	y_4	6	0	0	6	1	-2	3
-1	y_1	6	1	0	3	0	-1	-1
-2	y_2	2	0	1	-1	0	0	-1
	Z_j-C_j		0	0	2	0	1	1

Since the basic solution $x_1= 6$,$x_2 = 2$,$x_3= 0$ and Min Z =10 which is feasible so this is the required optimal solution.

XII Statement of the problem –IV

Use dual simplex method to solve the following linear programming problem:

Min Z = $6x_1 + 7x_2 + 3x_3 + 5x_4$
Subject to
$5x_1 + 6x_2 - 3x_3 + 4x_4 \geq 12$
$x_2 + 5x_3 - 6x_4 \geq 10$
$2x_1 + 5x_2 + x_3 + x_4 \geq 8$
$x_j \geq 0$

XIII Solution of the problem by Dual Simplex Method:

Max Z = $-6x_1 - 7x_2 - 3x_3 - 5x_4 + 0x_5 + 0x_6 + 0x_7$
Subject to
$-5x_1 - 6x_2 + 3x_3 - 4x_4 + x_5 = -12$
$x_2 + 5x_3 - 6x_4 + x_6 = -10$
$2x_1 + 5x_2 + x_3 + x_4 + x_7 = -8$
$x_j \geq 0$

Using Dual Simplex Method we get the basic solution $x_1=0$, $x_2=30/11$,$x_3=16/11$,$x_4=0$ and Min Z= 258/11 which is feasible so this is the required optimal solution.

XIV here we Apply Quick Simplex Method on Dual Simplex Method.

Step I Initial Table

C_B	Y_B	X_B	**-6**	**-7**	**-3**	**-5**	**0**	**0**	**0**
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
0	y_5	-12	-5	-6	3	-4	1	0	0
0	y_6	-10	0	-1	-5	6	0	1	0
0	y_7	-8	-2	**-5**	-1	-1	0	0	1
	Z_j-C_j		6	7	3	5	0	0	0
	$(Z_j$-$C_j)/y_{kj}$		**-6/5**	**-7/6**	**1**	**-5/4**			

Here we introduce y_2 , y_3 simultaneously and outgoing vectors are y_5 ,y_6.

To find new values in X_B column, we use the formulae from Quick Simplex Method, [7,8,9,10].

$$R = \begin{vmatrix} -6 & 3 \\ -1 & -5 \end{vmatrix} = 33 \quad C_1^{**} = \begin{vmatrix} -12 & 3 \\ -10 & -5 \end{vmatrix} / R = 30/11$$

$$C_2^{**} = \begin{vmatrix} -6 & -12 \\ -1 & -10 \end{vmatrix} / R = 16/11 , \qquad C_3^{**} = \frac{\begin{vmatrix} -6 & 3 & -12 \\ -1 & -5 & -10 \\ -5 & -1 & -8 \end{vmatrix}}{R} = 78/11$$

Since we get all the entries of X_B column positive so we can get the basic feasible solution. In this way we can find new values for each vector using Quick Simplex Method.

Step II Final Table

C_B	Y_B	X_B	-6	-7	-3	-5	0	0	0
			y_1	y_2	y_3	y_4	y_5	y_6	y_7
-7	y_2	30/11	25/33	1	0	2/33	-5/33	-1/11	0
-3	y_3	16/11	-5/33	0	1	-40/33	1/33	-2/11	0
0	y_7	78/11	18/11	**0**	0	-21/11	-8/11	-7/11	1
	Z_j-C_j		38/33	0	0	271/33	32/33	13/11	0

Using Quick Simplex Method we get the basic solution x_1=0, x_2=30/11, x_3=16/11, x_4=0 and Min Z= 258/11 which is feasible so this is the required optimal solution

7 Conclusion

It is observed that Quick Simplex Method is applicable to Simplex Method, Two Phase Method ,Games Problems [9-11]and from the above problems we conclude that it is effective in Dual Simplex Method also .Using Quick Simplex Method we get the solution in one step only. We have tried many problems with all the methods discussed and some of them are given as examples. It has been realized that quick simplex method helps in reducing number of iterations required to reach optimal solution in many problems. As far as calculation part is concerned finding determinant or basis inverse is not a very difficult job if computer is used.

Competing Interests

Author has declared that no competing interests exist.

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