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Analytical Modeling of the Thermoelectric Effect in Photovoltaic Cells: Combined Solar Photovoltaic and Thermoelectric Generator System (PV+TEG)

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Authors' contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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Abstract

Aims: Analytical modeling of the combined systems photovoltaic-thermoelectric (PV + TEG). The advantage of these systems is double:

- On the one hand, they allow to cool the photovoltaic cells (PV), which avoids the loss of electrical efficiency observed in the devices,
- On the other hand, recover this lost energy in the form of heat, and transform it into electrical energy thanks to the thermoelectric modules operating in Seebeck mode.

Study Design: Laboratory of Radiation Physics LPR, FAST-UAC, 01 BP 526, Cotonou, Benin.

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Methodology: We considered the temperature distribution in the semiconductor plate of the Thermoelectric Generator System (TEG). We resolved the thermal conductivity equation described by:

$$\frac{\partial T}{\partial t} - a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = Q(x, y, z,)$$

Where a^2 is the thermal diffusivity, Q(x, y, z) is the heat flow going from the PV to the TEG module which is dissipated through the latter; using constants variation method. We assumed that the temperature along the y-axis is considered uniform.

Results: The results obtained show that, the temperature distribution in the form of a traveling wave is maintained by external heating. This depends on both the hot and cold side temperature and the temperature span.

Conclusion: The heat flux available at the hot side of the TEG is assumed to be what remains of the absorbed radiation of the PV power production.

Keywords: Strong sunshine; quantum efficiency; thermoelectric module; Seebeck mode; temperature distribution; thermal conductivity.

1 Introduction

The major disadvantage of the photovoltaic cells is their low quantum efficiency. Indeed, only a small part of the power of the incident radiation is converted into electricity. Furthermore, unconverted radiations heat the photovoltaic panels, which further reduce their efficiency. Cooling systems are integrated at present in these cells, in order to decrease this temperature and allow a better quantum efficiency of the devices; hence the idea to realize the combined thermoelectric-photovoltaic systems [1].

Converting the energy from the Sun directly to electricity in an efficient way is of great interest. Photovoltaic devices (PV) can directly convert parts of the solar spectrum, but a significant part is absorbed as heat. In order to remedy this, a number of combined photovoltaic and heat recovery systems have been proposed recently. The most simple of these convert the heat energy directly to electrical energy using a thermoelectric generator (TEG). A number of such combined systems have been studied recently [2-7].

Coupled PV+TEG systems have also been considered solely from a modeling basis [8-16]. Experimental realizations of small combined PV and TEG systems have reported a significant increase in performance compared to that of a PV alone [17-26].

Compared with the single PV or TEG energy producing source, the (PV+TEG) hybrid energy source can offer some advantages, namely the better energy security due to the use of multiple energy sources, the higher fuel economy due to the increase of on-board renewable energy, and the higher control flexibility due to the coordination for charging the same pack of batteries. So the Thermoelectric-Photovoltaic hybrid energy source is promising for the application to HVs.

The performance of the combined system should be given in terms of both generated electric power and overall system efficiency by highlighting their dependence on environmental conditions, such as temperature and radiation, and on physical properties of the used material.

Starting from these considerations the principle of superposition is therefore usable; then the generated electrical power of the overall system will be the sum of the electric powers generated by both modules. Under this assumption the overall efficiency of the system can be calculated as the ratio between the sum of the generated electric powers by each module, and the power of the input system, i.e. the solar radiation available to the PV module.

Recently [27] has studied the performance of a combined solar photovoltaic (PV) and thermoelectric generator (TEG) system, using an analytical model for four different types of commercial PVs and a commercial bismuth telluride TEG. The TEG is applied directly on the back of the PV, so that the two devices have the same temperature. The PVs considered are crystalline Si (c-Si), amorphous Si (a-Si), copper indium gallium (di)selenide (CIGS) and cadmium telluride (CdTe) cells. They have shown that, the degradation of PV performance with temperature is shown to dominate the increase in power produced by the TEG, due to the low efficiency of the TEG. For c-Si, CIGS and CdTe PV cells the combined system produces a lower power and has a lower efficiency than the PV alone, whereas for an a-Si cell the total system performance may be slightly increased by the TEG.

More recently [28] presents a dynamic theory of the thermoelectric effect that occurs in a uniform unipolar semiconductor under the influence of a time-dependent temperature distribution. To be specific, the temperature distribution is chosen as a plane traveling wave of constant amplitude. The dependence of the thermoelectric current, appearing under these conditions, on the semiconductor properties, as well as on the parameters of the temperature wave, is studied.

In this paper, we considered the temperature distribution in the semiconductor plate of the Thermoelectric Generator System (TEG). We resolved the thermal conductivity equation described by:

$$\frac{\partial T}{\partial t} - a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = Q(x, y, z, t) \quad (1)$$

Where a^2 is the thermal diffusivity, Q(x, y, z, t) is the heat flow going from the PV to the TEG module which is dissipated through the latter; using constants variation method. We assumed that the temperature along the y-axis is considered uniform.

The paper is organized as follows. In Section 2, the combined PV and TEG system considered is presented. Next, the theoretical approach is established. In Section 3, the solving of the thermal conductivity equation is described. Finally, the conclusion and outlook are listed in Section 4.

2 Materials and Methods

2.1 The studied system

We consider a combined (PV+TEG) system, where the TEG is mounted directly on the back of the PV. The hot side temperature of the TEG is thus equal to the temperature of the PV, $T_{PV} = T_{TEG.hot}$. The system is shined upon by the Sun, with an arbitrary concentration of the light. The studied setup is illustrated in Fig. 1.

We assume that the properties of the PV do not change with solar concentration, but only with temperature. We note that the cold side of the TEG is assumed to be perfectly cooled to the reference temperature 25°C in the remainder of this paper. The system operates at room temperature having as input the solar radiation and as output the total electric power generated by the system. At high solar irradiance the PV module temperature (T_{max}) can reach 50-80°C. These values depend on the site, the type of the integration and of the period of year considered.

We assume that the photons provided from the Sun are either converted directly into electrical energy, with fraction η_{PV}, converted into heat, η_T, or not absorbed, η_{non}, such that

$$\eta_{PV} + \eta_T + \eta_{non} = 100\% \quad (2)$$

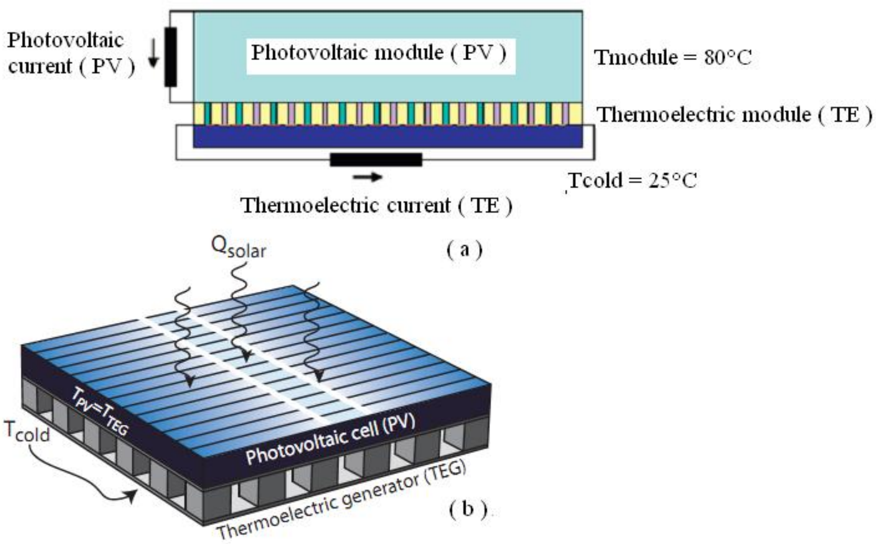

Fig. 1. An illustration of the system considered. a- Equivalent system; current generator, b- the incoming solar radiation hits the PV, which heats up from parts of the radiation. This heat is transferred through the TEG, to the cold side [27]

The fraction of non-absorbed photons is taken to be constant.

$$\eta_{PV} = \eta_{T_{ref}}\left(1 + \beta\left(T_{PV,0} - T\right) \right) \quad (3)$$

where $\eta_{T_{ref}}$ is the reference temperature, typically 25°C, and β is the temperature coefficient [29-30].

The efficiency of a TEG is generally a nonlinear function of temperature since the thermoelectric material properties vary nonlinearly with temperature. In the case when the material properties do not depend on temperature an analytic expression for the efficiency can be derived. This depends on both the hot and cold side temperature and the temperature span. However, this assumption is not applicable for actual thermoelectric materials [27].

The heat flux available at the hot side of the TEG is assumed to be what remains of the absorbed radiation of the PV power production:

$$Q_{TEG}(T) = Q_{Solar}(1 - \eta_{PV}(T) - \eta_{non}) \quad (4)$$

The total electrical power produced is the sum of the power produced by the PV and by the TEG:

$$P_{tot}(T) = Q_{Solar}\eta_{PV}(T) + Q_{TEG}(T)\eta_{TEG}(T) \quad (5)$$

The maximum total power produced by the combined PV and TEG system can then be obtained. The efficiency of a coupled PV and TEG system are given as.

$$\eta_{PV+TEG}(T) = \eta_{PV}(T) + \eta_{TEG}(T)(1 - \eta_{PV}(T) - \eta_{non}) \quad (6)$$

An amorphous Si (a-Si) solar photovoltaic (PV) and a commercially available bismuth telluride (CdTe) (TEG) is the combined (PV+TEG) system considered in this study.

2.2 Method: Boundary conditions (BCs)

The Boundary conditions (BCs) for equation (1) in the geometry under consideration are as follows. If the upper surface of the sample (Z = 0) is illuminated with light of intensity $I = I_0(1 + cos\emptyset)$, completely absorbed in an infinity thin near-surface layer of the sample without photocarrier generation, the appropriate (BCs) are as follows:

$$\left.\frac{\partial T}{\partial z}\right|_{Z=0} = -\frac{g_0 I_0}{\kappa}\ (1 + cos\emptyset) \tag{7}$$

where: the thermal conductivity of the semiconductor, g_0: the heating efficiency of the semiconductor by the light.

On the opposite side of the sample (z = h), which is the interface with the substrate serving as the thermostat, the BC is written in the form [28].

$$\left.\frac{\partial T}{\partial z}\right|_{Z=h} = -\frac{\chi}{\kappa}\ (T - T_0) \tag{8}$$

where χ : is the surface thermal conductivity.

Along the x axis, we have:

$$\left.\frac{\partial T}{\partial z}\right|_{Z=0} = \left.\frac{\partial T}{\partial z}\right|_{Z=h} \tag{9}$$

These boundary conditions are exploited to solve the equation (1).

3 Results and Discussion

3.1 Resolution of homogeneous equation

The temperature homogeneous distribution in the combined (PV+TEG) system is described by the thermal conductivity equation (10).

$$\frac{\partial T}{\partial t} - a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = 0 \tag{10}$$

The homogeneous solution of equation (10) will be expressed in the form of

$$T(x, z, t) = A_1(z)sin\Phi + A_2(z)cos\Phi + A_3(z) \tag{11}$$

Substituting equation (11) in (equation (1), one obtains the system of equations for the temperature wave amplitudes $A_1(z)$ - $A_3(z)$.

$$A_1''(z) - k^2 A_1(z) + \frac{\omega}{a^2} A_2(z) = 0 \tag{12}$$

$$A_2''(z) - k^2 A_2(z) - \frac{\omega}{a^2} A_1(z) = 0 \tag{13}$$

$$A_1''(z) = 0 \tag{14}$$

with boundary condition (BC), we have:

$$A_1'(0) = 0, A_2'(0) = A_3'(0) = -\frac{g_0 I_0}{k} \tag{15}$$

$$A_{1,2}'(h) = -\frac{\chi}{k} A_{1,2}(h) \tag{16}$$

$$A_3'(h) = -\frac{\chi}{k}(A_3(h) - T_0) \tag{17}$$

The general solution to the system of equations (12)-(14) has the following form:

$$A_1(z) = \frac{C_1 + iC_3}{2} cosh\frac{k_1 z}{a} + \frac{C_1 + iC_3}{2} cosh\frac{k_2 z}{a} + \frac{a(C_2 + lC_4)}{2k_1} sinh\frac{k_1 z}{a} + \frac{a(C_2 + lC_4)}{2k_2} sinh\frac{k_2 z}{a} \tag{18}$$

$$A_2(z) = \frac{C_3 + lC_1}{2} cosh\frac{k_1 z}{a} + \frac{C_3 + lC_1}{2} cosh\frac{k_2 z}{a} + \frac{a(C_4 + lC_2)}{2k_1} sinh\frac{k_1 z}{a} + \frac{a(C_4 + lC_2)}{2k_2} sinh\frac{k_2 z}{a}, \tag{19}$$

$$A_3(z) = C_5 z + C_6 \tag{20}$$

Where $k_1^2 = a^2k^2 + i\omega$; $k_2^2 = a^2k^2 - i\omega$ C_{1-6} are the integration constants.

When $\frac{h}{a}\left|k_{1,2}\right| \ll 1$ (For a thermally thin sample), equation (18) and equation (19) become:

$$A_1(z) = C_1 + C_2 z \quad \text{and} \quad A_2(z) = C_3 + C_4 z \tag{21}$$

Substituting equation (20) and equation (21) in equation (15), one obtains:

$$C_2 = 0 \ \text{ and } \ C_4 = C_5 = -\frac{g_0 I_0}{k} \tag{22}$$

The integration constants C_1 and C_2 are calculated, substituting equation (21) in equation (16), we obtain:

$$C_1 = 0 \ \text{ and } C_3 = \frac{g_0 I_0 (k + \chi h)}{k\chi} \tag{23}$$

C_6 is determined, substituting equation (20) in equation (17). We have:

$$C_6 = T_0 - \frac{g_0 I_0 (k + \chi h)}{k\chi} \tag{24}$$

Once the integration constants have been determined, we have substituted equation (20) and equation (21) in equation (11), and we obtained the following temperature distribution in the thermally thin sample by:

$$T(x,z,t) = T_0 - \frac{g_0 I_0}{k}\left(z - \frac{\kappa + \chi h}{\chi}\right)(1 + cos\Phi) \tag{25}$$

When the semiconductor is weakly coupled to the substrate, we have $(\chi h \ll \kappa)$, and the temperature distribution is given as follows:

$$T(x,z,t) = T_0 + \frac{g_0 I_0}{k}(1 + cos\Phi) \tag{26}$$

3.2 Resolution of inhomogeneous equation

The temperature inhomogeneous distribution in the combined (PV+TEG) system is described by the thermal conductivity equation

$$\frac{\partial T}{\partial t} - a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right) = Q(x, y, z, t) \tag{27}$$

where $Q(x, y, z, t) = G_0 e^{-\beta z} + I_0(1 + cos\Phi)$:

The general form of the particular solution is given by:

$$Q(x, y, z, t) = A(z)e^{-\beta z} + B(z)sin\Phi + C(z)cos\Phi + D(z) \tag{28}$$

We have sought the constants A(z), B(z), C(z) and D(z) by the following transformations:

$$\frac{\partial Q}{\partial t} = wB(z)cos\Phi - wc(z)sin\Phi \tag{29}$$

$$\frac{\partial Q}{\partial x} = kB(z)cos\Phi - kC(z)sin\Phi \tag{30}$$

$$\frac{\partial^2 Q}{\partial x^2} = -k^2 B(z)sin\Phi - k^2 C(z)cos\Phi$$

$$\frac{\partial^2 Q}{\partial y^2} = -0 \tag{31}$$

$$\frac{\partial Q}{\partial y} = A^{'}(z)e^{-\beta z} - A(z)\beta e^{-\beta z} + B^{'}(z)sin\Phi - C^{'}(z)cos\Phi + \mathrm{D}^{'}(\mathrm{z}) \tag{32}$$

$$\frac{\partial^2 Q}{\partial z^2} = A^{''}(z)e^{-\beta z} - A^{'}(z)\beta e^{-\beta z} - A^{'}(z)\beta e^{-\beta z} - A(z)\beta^2 e^{-\beta z} + B^{''}(z)sin\Phi - C^{''}(z)cos\Phi + \mathrm{D}^{''}(\mathrm{z}) \tag{33}$$

$$\frac{\partial^2 Q}{\partial z^2} = (A^{''}(z) - 2A^{'}(z)\beta - A(z)\beta^2)e^{-\beta z} + B^{''}(z)sin\Phi - C^{''}(z)cos\Phi + \mathrm{D}^{''}(\mathrm{z}) \tag{34}$$

By identification, we have:

$$A^{''}(z) - 2A^{'}(z)\beta - A(z)\beta^2 = G_0 \tag{35}$$

$$\mathrm{B}^{''}(\mathrm{z}) - k^2 B(z) + \frac{w}{a^2}C(z) = I_0 \tag{36}$$

$$\mathrm{C}^{''}(\mathrm{z}) - k^2 C(z) - \frac{w}{a^2}B(z) = 0 \tag{37}$$

$$D^{''}(z) = -\frac{I_0}{a^2} \tag{38}$$

$$D^{''}(z) = -\frac{I_0}{a^2} \quad \text{and} \quad D^{'}(z) = -\frac{I_0}{a^2}z + \delta \tag{39}$$

$$D(z) = -\frac{I_0}{2a^2}z^2 + \delta z + \gamma \tag{40}$$

We determine the constants $\delta\ et\ \gamma$, using the boundary Conditions (15) and (17):

$$D^{'}(z) = -\frac{I_0}{a^2}z + \delta \quad \text{therefore} \quad D^{'}(0) = \delta$$
$$D^{'}(0) = \delta = -\frac{g_0 I_0}{k} \tag{41}$$

We posed:

$$D^{'}(h) = -\frac{\chi}{k}[D(h) - T_0] = D(z) = -\frac{\chi}{k}\left[-\frac{I_0}{2a^2}h^2 - \frac{g_0 I_0}{k}h + \gamma - T_0\right] \tag{42}$$

That to say $$D^{'}(h) = -\frac{I_0}{a^2}h - \frac{g_0 I_0}{k} \tag{43}$$

By combining equation (42) and equation (43), we have:

$$-\frac{\chi}{k}\left[-\frac{I_0}{2a^2}h^2 - \frac{g_0 I_0}{k}h + c - T_0\right] = -\frac{I_0}{a^2}h - \frac{g_0 I_0}{k} \tag{44}$$

Then $$\gamma = T_0 + \frac{I_0}{2a^2}h^2 + \frac{g_0 I_0}{k}h + \frac{I_0}{\chi a^2}h + \frac{g_0 I_0}{\chi} \tag{45}$$

And we obtain:

$$D(z) = -\frac{I_0}{2a^2}z^2 - \frac{g_0 I_0}{k}z + \left(T_0 + \frac{I_0}{2a^2}h^2 + \frac{g_0 I_0}{k}h + \frac{I_0}{\chi a^2}h + \frac{g_0 I_0}{\chi}\right) \tag{46}$$

The constant B (z) is given by:

$$B(z) = \frac{+iC_3}{2}cosh\frac{k_1 z}{a} + \frac{C_1 - iC_3}{2}cosh\frac{k_2 z}{a} + \ \frac{C_2 + iC_4}{2k_1}cosh\frac{k_1 z}{a} + \frac{C_2 - iC_4}{2k_2}cosh\frac{k_2 z}{a} \tag{47}$$

We search now C (z):

We identified C''(z) to I_o from (equation (37).

$$C^{''} = -\ I_o \tag{48}$$

$$C^{'}(z) = -I_0 z + \varepsilon \tag{49}$$

$$C(z) = \frac{-I_0}{2}z^2 + \varepsilon z + \vartheta \tag{50}$$

where $C^{'}(0) = \varepsilon = -\frac{g_0 I_0}{k}$

as far as $$C^{'}(h) = -\frac{\chi}{k}C(h) = -I_0 h - \frac{g_0 I_0}{k} = -\frac{\chi}{k}\left(-\frac{I_0}{2}h^2 + \varepsilon h + \vartheta\ \right) \tag{51}$$

$$-\frac{I_0}{2}h^2 + -\frac{g_0 I_0}{k}h + \vartheta = I_0\frac{k}{\chi}h + \frac{g_0 I_0}{\chi} \tag{52}$$

and

$$\vartheta = I_0\frac{k}{\chi}h + \frac{g_0 I_0}{\chi} + -\frac{I_0}{2}h^2 + \frac{g_0 I_0}{k}h \tag{53}$$

$$C(z) = \frac{-I_0}{2}z^2 - \frac{g_0 I_0}{k}z + I_0\frac{k}{\chi}h + \frac{g_0 I_0}{\chi} + -\frac{I_0}{2}h^2 + \frac{g_0 I_0}{k}h \tag{54}$$

We search A (z):

$$A''(z) - 2A'(z)\beta - A(z)\beta^2 = G_0 \tag{55}$$

One poses $y'' = A''(z)$; $y' = A'(z)$ et $y = A(z)$

we have $y'' - 2\beta y' - \beta^2 y = 0$ (56)

The characteristic equation of equation (56) is given by:

$$r^2 - 2\beta r - \beta^2 = 0 \tag{57}$$
$$\Delta = 4\beta^2 - 4\beta^2 = 0$$
$$r = \beta$$

The homogeneous solution of equation (56) is given by

$$y_h = (C_1 + C_2 z)e^{\beta z} \tag{58}$$

The particular solution of equation (55) is:

$$y_p = \frac{G_0}{\beta^2} \tag{59}$$

General solution of equation (55) is expressed by:

$$y = y_h + y_p = (C_1 + C_2 z)e^{\beta z} + \frac{G_0}{\beta^2} \tag{60}$$

Identifying y to A(z), we have:

$$A(z) = (C_1 + C_2 z)e^{\beta z} + \frac{G_0}{\beta^2} \tag{61}$$

where $e^{\beta z} = \cosh \beta z + \sin \beta z$

Therefore $A(z) = (C_1 + C_2 z)\,(\cosh \beta z + \sin \beta z) + \frac{G_0}{\beta^2}$ (62)

The general solution of equation (1) becomes:

$$T(x,z,t) = T_0 - \frac{g_0 I_0}{k}\left(z - \frac{\kappa + \chi h}{\chi}\right)(1 + cos\Phi) + A(z)e^{-\beta z} + B(z)sin\Phi + C(z)cos\Phi + D(z) \tag{63}$$

For $\chi h \ll \kappa$

$$T(x,z,t) = T_0 + \frac{g_0 I_0}{k}(1 + cos\Phi) + A(z)e^{-\beta z} + B(z)sin\Phi + C(z)cos\Phi + D(z) \tag{64}$$

where the constants $A(z)$, $B(z)$, $C(z)$ and $D(z)$ are determined above.

3.3 Maximum temperature of combined (PV + TEG) system

The efficiency as a function of temperature span is given by [20].

$$\eta_{TEG} = \alpha \Delta T^2 + \delta \Delta T \tag{65}$$

Where $\alpha = -1.21(5) * 10^{-6} K^{-2}$ and $\delta = 4.87(7) * 10^{-4} K^{-1}$ for a cold side temperature of 25°C.

From equation (65), equation (4) and equation (5), we deducted the temperature at which the total power output is maximum. This is given by [27].

$$T_{max} = \frac{1}{4\alpha\beta\eta_{PV,o}}(\pm 2T_C\alpha\beta\eta_{PV,o} \pm 2T_{ref}\alpha\beta\eta_{PV,o} \pm \beta\delta\eta_{PV,o} \pm 2\alpha\eta_{non} \pm 2\alpha\eta_{PV,o} \pm 2\alpha \pm \left(4{T_C}^2\alpha^2\beta^2(\eta_{PV,o})^2 - 8T_C * T_{ref}\alpha^2\beta^2(\eta_{PV,o})^2 + 4{T_{ref}}^2\alpha^2\beta^2(\eta_{PV,o})^2 - 4T_C\alpha\beta^2\delta(\eta_{PV,o})^2 + 4T_{ref}\alpha\beta^2\delta(\eta_{PV,o})^2 - 8T_C\alpha^2\beta\eta_{non}\eta_{PV,o} - 8T_C\alpha^2\beta(\eta_{PV,o})^2 + 8T_{ref}\alpha^2\beta\eta_{non}\eta_{PV,o} + 8T_{ref}\alpha^2\beta(\eta_{PV,o})^2 + \beta^2\delta^2(\eta_{PV,o})^2 + 8T_C\alpha^2\beta\eta_{PV,o} - 8T_{ref}\alpha^2\beta\eta_{PV,o} + 8\alpha\beta^2(\eta_{PV,o})^2 + 4\alpha\beta\delta(\eta_{PV,o})^2 + 4\alpha^2(\eta_{non})^2 + 8\alpha^2\eta_{non}\eta_{PV,o} + 4\alpha^2(\eta_{PV,o})^2 - 4\alpha\beta\delta\eta_{PV,o} - 8\alpha^2\eta_{non} - 8\alpha^2\eta_{PV,o} + 4\alpha^2\right)^{\frac{1}{2}}) \quad (66)$$

The total power produced by the (PV +TEG) system can then be found using equation (5) and the solution for the optimum temperature given in the expression above equation (66).

4 Conclusions and Outlook

The performance of a combined solar photovoltaic (PV) and thermoelectric generator (TEG) system is examined, using an analytical model to study the temperature distribution in the semiconductor plate of the Thermoelectric Generator System (TEG). An amorphous Si (a-Si) solar photovoltaic (PV) and a commercially available bismuth telluride (CdTe) (TEG) is the combined (PV+TEG) system considered in this work.

We resolved the thermal conductivity equation, using constants variation method. We assumed that the temperature along the y-axis is considered uniform.

Our results reveal that, the temperature distribution in the form of a traveling wave is maintained by external heating. This depends on both the hot and cold side temperature and the temperature span.

The heat flux Q(x, y, z, t) available at the hot side of the TEG is assumed to be what remains of the absorbed radiation of the PV power production. The efficiency is directly proportional to the produced power, with the proportionally factor being Q_{solar}. The maximum operating temperature of a BiTe TEG is indicated.

Future work can be about:

1) - Extensions to the model;
2) - Improvement of the analytical results;
3) - Compare experimental results with those obtained analytically.

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Competing Interests

Authors have declared that no competing interests exist.

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