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Short Note on Kyle's Equilibrium Class

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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Short Research Article

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Abstract

The asymmetric information plays critical role in all economics. In the presence of asymmetric information in a given market, market prices of assets are different with those prices under the no arbitrage assumption. It has fundamental effects on the market equilibrium. [1] considered three types of traders: noise trader, informed trader and market maker in a given market in the presence of asymmetric information property. He derived the equilibrium prices of assets. In this short note, Kyle's results are extended. It is seen that a class of equilibrium prices exists, referred as the Kyle's equilibrium class. To this end, first, it is proved that there is a simple linear relation between the variance of equilibrium price and the variance of traded asset size. Then, this simple relation is replaced with a general linear relation. By maximizing the profit function of informed trader, in this case, the Kyle's equilibrium class is derived. Simulation results are also given. Finally, a conclusion section is given.

Keywords: Asymmetric information; class of equilibrium; Kyle's Equilibrium; normal distribution.

1 Introduction

Information is an important concept in all economics. It plays important role in determining the market prices of assets. When a market is complete and the no arbitrage assumption exists, all available information

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is reflected by prices and financial market is reached to equilibrium. Before this step, informed traders gain profit in the market. Indeed, asymmetric information phenomena occurs if, in a special market, some agents have special information while the others do not have (see [2]). [1] studied the effect of asymmetric information on market price of a risky asset. He assumed that three types of traders including informed trader, noise trader and market maker exist in the market. Indeed, he proposed a model for financial asset pricing under asymmetric information which is totally different with market prices under the no arbitrage assumption.

The model of Kyle has been extended in various ways. [3,4] studied the effect of asymmetric information in option pricing problems. [5] extended the work of [3] to the case of continuous time market model. [6] studied the problem of quadratic hedging under the Kyle model. [7] introduced the concept of emergence of probability to propose the game theoretic aspects of Kyle's model. [8] studied the completeness of market and robust pricing for Kyle's problem. [9] proposed a robust framework for quantifying the value of information in pricing and hedging.

In this paper, a new class of Kyle equilibrium is derived. First, his results are extended. It is proved that there is a simple linear relation between the variance of risk neutral price and the variance of size of asset traded by noise trader. Then, the variance of price and the variance of size are transformed by scale factors α^2 and β^2, respectively. By maximizing the profit of informed trader with respect to α and β, the new Kyle's equilibrium price is derived. It is a function of α and β which defines a class for equilibrium prices. It is refereed as the Kyle's equilibrium class. The rest of paper is organized as follows. In the next section, the main theoretical results are given. Some simulation results are given in section 3. Conclusions are given in section 4.

2 Main Results

Using the Kyle's notations, the price of asset is V which is normally distributed with mean μ_v and σ_v^2, that is $V \sim \mathrm{N}(\mu_v, \sigma_v^2)$, where U and V are independently distributed. The noise trader trades $\mathrm{U} \sim \mathrm{N}(0, \sigma_\mathrm{u}^2)$ size of asset and the informed trader decisions about $X = \mathrm{aV} + \mathrm{b}$ size of asset, where under equilibrium case, then

$$a = \frac{\sigma_u}{\sigma_V} \text{ and } b = -\frac{\sigma_u}{\sigma_V}\, \mu_v.$$

The price maker suggests the risk neutral price is $p = p(y) = E(V|Y = y) = cy + d$, where under equilibrium condition, then $c = \frac{\sigma_V}{2\sigma_u}$, $d = \mu_v$ and $Y = X + U$.

2.1 Some results

Notice that $E(X) = 0, var(X) = var(U) = \sigma_u^2$. Thus, it is easy to see that $E(Y) = 0$ and $var(Y) = 2\sigma_u^2$. Also, $E\big(p(Y)\big) = \mu_v$ and $var\big(p(Y)\big) = 0.5\sigma_u^2$. The following proposition states that the equilibrium conditions are derived using assuming simple conditions like $var(X) = \sigma_u^2$ and $var\big(p(Y)\big) = 0.5\sigma_u^2$. Also, the statistical distribution of informed trader profit is derived.

Proposition 1. If $E(X) = 0, var(X) = \sigma_u^2$ and $E\big(p(Y)\big) = \mu_v$ and $var\big(p(Y)\big) = 0.5\sigma_u^2$, then

(a) The coefficients of equilibrium p and X is obtained.
(b) The profit $\pi = -\frac{\sigma_u}{\sigma_V}(\mathrm{V} - \mu_v)^2$ is distributed as

$$-\sigma_u\sigma_v\chi^2_{(1)},$$

with mean and variance $-\sigma_u\sigma_v$, $2\sigma_u^2\sigma_v^2$, respectively.

2.2 Motivation

This fact motivates author to let similar conditions and to derive the class of equilibrium conditions. As follows, the class of equilibrium cases is characterized. To this end, suppose that

$$E(X) = 0, var(X) = \alpha^2\sigma_u^2,$$

and

$$E\big(p(Y)\big) = \mu_v\,,\, var\big(p(Y)\big) = \beta^2\sigma_u^2,$$

for some real numbers α and β.

Proposition 2. Under the new equilibrium conditions, i.e., $var(X) = \alpha^2\sigma_u^2$ and $var\big(p(Y)\big) = \beta^2\sigma_u^2$, then

(a) $a = \alpha\frac{\sigma_u}{\sigma_v}$, $b = -a\mu_v$, $c = \frac{\beta}{\sqrt{1+\alpha^2}}$ and $d = \mu_v$.
(b) $\pi = a(1 - ac)(\mathrm{V} - \mu_v)^2 = \alpha\left(1 - \frac{\sigma_u}{\sigma_v}\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\right)\sigma_u\sigma_V Z^2$, where Z is standard normal random variables.

2.3 Results

Here, the profit π is maximized with respect to α and β. To this end, let

$$f(\alpha,\beta) = \alpha\left(1 - \frac{\sigma_u}{\sigma_v}\frac{\alpha\beta}{\sqrt{1+\alpha^2}}\right).$$

Then, $\frac{\partial f}{\partial \alpha} = 0$ implies that

$$\beta = \frac{(1+\alpha^2)^{\frac{3}{2}}}{(\alpha^2+2)\alpha}\frac{\sigma_V}{\sigma_u}.$$

This is the main equation of this paper. The above results are summarized in the following theorem.

Theorem 1. The new class of Kyle equilibrium is given by

$$\begin{cases} p = p(y) = E(V|Y = y) = cy + d, \\ \qquad Y = X + U, \\ \qquad \beta = \dfrac{(1+\alpha^2)^{\frac{3}{2}}}{(\alpha^2+2)\alpha}\dfrac{\sigma_V}{\sigma_u}, \end{cases}$$

where for suitable choice of α, corresponding values for β's are given. Here, $c = \frac{\beta}{\sqrt{1+\alpha^2}}$ and $d = \mu_v$.

Remark 1. As special case, suppose that $= \sqrt{1+\alpha^2}$. Then,

$$\pi = kf(\alpha) = \alpha\left(1 - \frac{\sigma_u}{\sigma_v}\alpha\right),$$

where k is positive number. This function has maximum at $\alpha = \frac{\sigma_V}{2\sigma_u}$. Then, $a = 0.5, b = -0.5\mu_v, c = 1$ and $d = \mu_v$. By this selection, then $f(\alpha, \beta) = \frac{\alpha}{\alpha^2+2}$. Then, $\frac{\partial f}{\partial \alpha} = 0$ makes the optimum α is $\sqrt{2}$. In this way, $\beta = \sqrt{\frac{27}{32}}\frac{\sigma_V}{\sigma_u}$.

3 Simulations

The results of previous sections are derived by normality assumptions on U and V. In this section, these assumptions are relaxed and similar results are derived using a simulation-based approach. To this end, let,

$$Y = X + U, X = \mathrm{a}V + \mathrm{b},$$
$$a = \alpha\frac{\sigma_u}{\sigma_v}, b = -a\mu_v, ,$$
$$p = p(y) = E(V|Y = y),$$
$$var(p(Y)) = \beta^2\sigma_u^2,$$
$$V\sim(\mu_v, \sigma_v^2), \mathrm{U}\sim(0, \sigma_\mathrm{u}^2),$$

where U and V are independently distributed. Many different statistical distributions can be selected for U and V. Here, for example, let U has uniform distribution on (-1,1), then $\sigma_\mathrm{u}^2 = \frac{1}{3}$ and V has gamma distribution $Gam(1,2)$. Then, $\mu_v = 2$ and $\sigma_v^2 = 4$. Let $\alpha = 1$. Then, $a = \frac{\sqrt{3}}{6}$ and $b = -\frac{\sqrt{3}}{3}$. Here, $var(p(Y))$ is approximated. To this end, $p = E(V|Y = y)$ is approximated by a Monte Carlo method with 1000 repetitions. Indeed, 1000 samples of V and U are generated Y is computed. Simulated values of V and Y are categorized to 996 categories where category $i -$ th contains $V_k, Y_k, k = i, \dots, i + 4$ observations. Here, the idea of the moving average filtering technique is applied. The sample mean of $Y_k, k = i, \dots, i + 4$ is considered as y_i and the sample mean of $V_k, k = i, \dots, i + 4$ plays the role of $E(V|Y = y_i)$, i.e., p_i. The regression between y_i and p_i gives the functional form of $p = E(V|Y = y)$. The following figure shows the scatter plot of p_i against y_i.

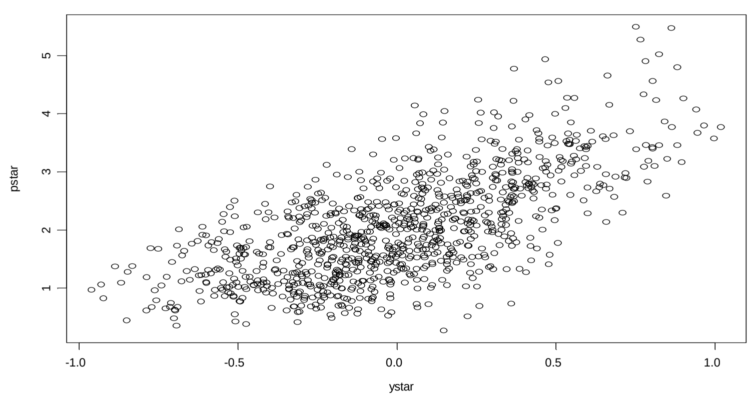

Fig. 1. Scatter plot of pi versus yi

The regression line fitted to this data set $p(y) = 1.68y + 2.038$. Therefore, $\beta = \sqrt{\frac{var(p(Y))}{\sigma_u^2}} = 1.57$. This situation is equivalent to normal case with $\alpha = 1$ and $\beta = 1.57$.

4 Conclusions

Although, to the best knowledge of author, this paper is the first attempt to extend the Kyle result to the class of Kyle equilibrium, however, in this section, as a referee suggested, the results obtained in the previous sections are put briefly in relation to the literatures produced after Kyle [1]. Kyle [1] constructed a model to exploit monopoly power in the market. His work is extended in two ways. Some of researchers such as Duffie (1985), Back [3], Campi [6], Aksamit and Hou [9] extended the problem proposed by Kyle by changing his model. For example, they considered the Black-Scholes model of asset pricing for traders. Some others like Back [4] applied the model of Kyle for financial modeling such as option pricing.

Competing Interests

Author has declared that no competing interests exist.

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